# Week 4

2.77

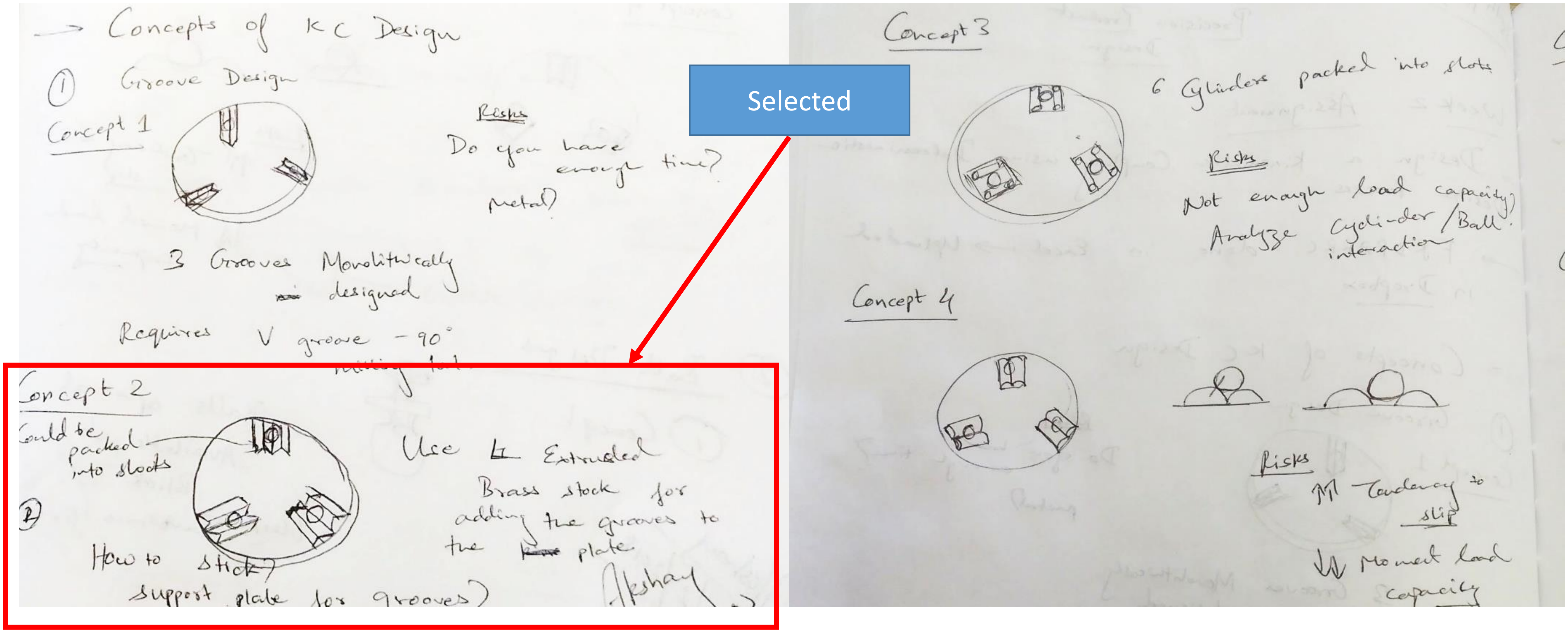
Akshay Harlalka

# Kinematic Coupling

# FRDPPARC for Kinematic Coupling

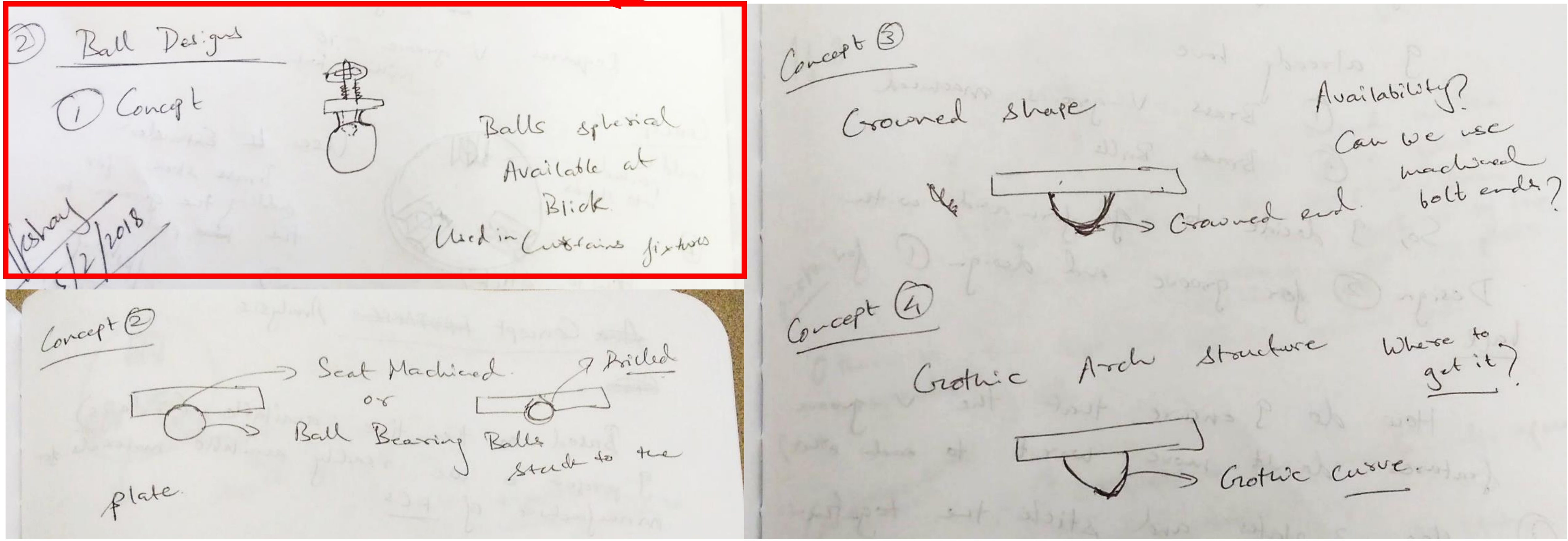
| Sr.No | Functional Requirements | Design Parameters | Analysis | References | Risks | Counter Measures |
|---|---|---|---|---|---|---|
| 1 | Ability to bear reasonable Loads/Moments | *Axial Load Capacity* | Hertz Contact Stress calculations | | | |
| | | a. Groove type - (V groove, cylindrical, or other parametric curve) | Material yields? | KC Design Document | Material yields and KC deforms permanently under load | Add a reasonable safety factor to calculations |
| | | b. Groove Material - (Steel, Wood, Brass, Aluminium, Sheet Metal, Plastic) | If using adhesive/epoxy, analyse if sufficient bonding strength is present to resist loads and moments? | Book - Precision Machine Design by Slocum | | |
| | | c. Ball type - (Spherical, Cone, Gothic) | | Chapter 9 of Fundamentals book | | |
| | | d. Ball Material - (Steel, Plastic, Wood, Brass) | | | | |
| | | | | | | |
| 2 | Repeatability | Ball/Groove Material | Relative hardness | Book - Precision Machine Design by Slocum | Materials corrode and repeatability is lost | Select non-fretting materials for ball and grooves |
| | | | | | | |
| 3 | Stiffness | Stiffness of Materials for ball/groove | Young's Modulus of the materials<br><br>Flexural stiffness of the groove if using non-monolithic features | KC Design Spreadsheet | Accuracy will not be good | High stiffness materials |
| | | Contact angle | Contact Angle should be at 45 deg for balanced stiffness | | | |
| | | | | | | |
| 4 | Accuracy | Effect of Manufacturing tolerances | Use worst case analysis of the how the errors in manufacturing can cause tilt/parasitic errors | Precision machine Design Book | | |
| | | Deflections | Find deflections and analyze/predict the accuracy | KC Design Spreadsheet | | |
| | | | | | | |
| 5 | Ease of Manufacture | What do I have? | Resource Assesment | MW Website, Hobby Shop site | I may not finish the KC by the deadline. --Bad | Work fast and use readily available materials |
| | | Time Constraints | Bank Balance | | | |
| | | Budget Constraints | Time available - | | | |
| | | Accessibility to machine tools | Machine Shop Schedules | | | |

# Concept Generation - Grooves



# Concept Generation - Balls

Selected




**The critical factor which tilted the balance in the favour of these concepts was ease of manufacturing and the availability of components required for the fabrication.**

# Analysis – Estimating Deflection under the balls and Axial Stiffness

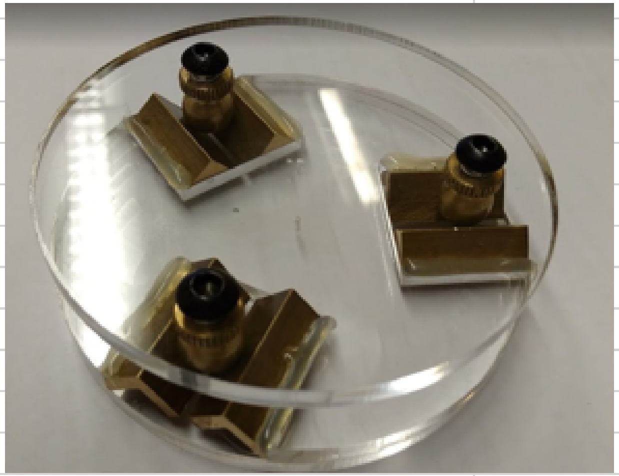
*Kinematic Coupling Design Spreadsheet - Akshay Harlalka 17/02/2018*

*For this specific case, major and minor dia for both groove and ball is same*

**Predicting Deflection and Contact Stress using Hertz Contact Stress Theory**

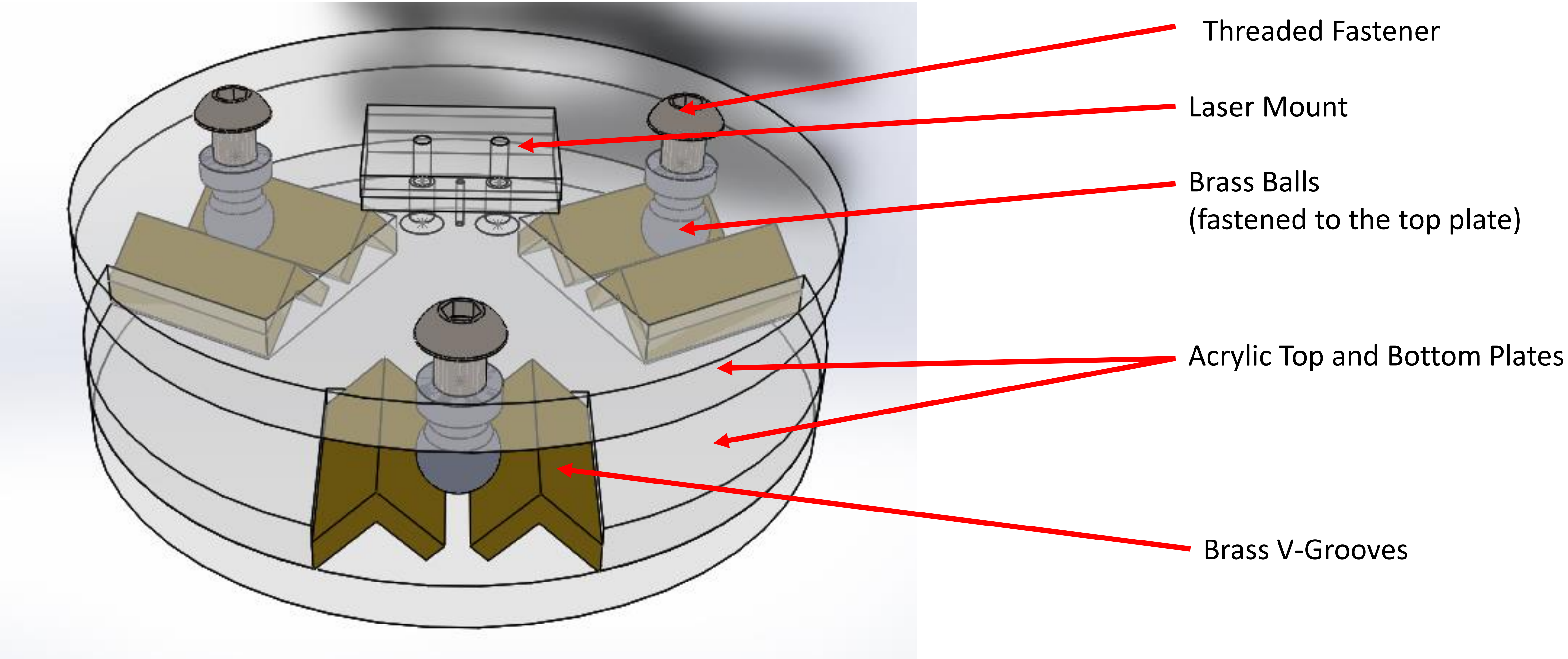
| Parameter | Value | Units |
|---|---|---|
| Radius of the ball (rb) | 5 | mm |
| Radius of groove (rg) | 1.E+08 | mm |
| | | |
| Equivalent Radius (Re) | 2.5 | mm |
| Equivalent Young's Modulus (Ee) | 5.4E+04 | Mpa |
| Costheta | 0.00 | |
| Theta | 1.57 | Radians |
| Alpha | 1.00 | |
| Beta | 1.00 | |
| Lambda | 0.74 | |
| Contact Area Elliptical SemiAxes (c) | 0.05 | mm |
| Contact Area Elliptical SemiAxes (d) | 0.05 | mm |
| Contact Pressure (q) | 3.2E+02 | N/mm2 |
| Contact Stress Ratio | 0.5 | |
| Deflection under the balls | 0.4 | um |
| Stiffness | 28.6 | N/um |

| Data | Brass | Units |
|---|---|---|
| Young's Modulus | 9.7E+04 | Mpa |
| Poissons Ratio | 0.31 | |
| Ultimate Tensile Strength | 400 | Mpa |
| Force Applied on center of coupling | 12.3 | N |
| Contact Force at each interface | 1.45 | N |

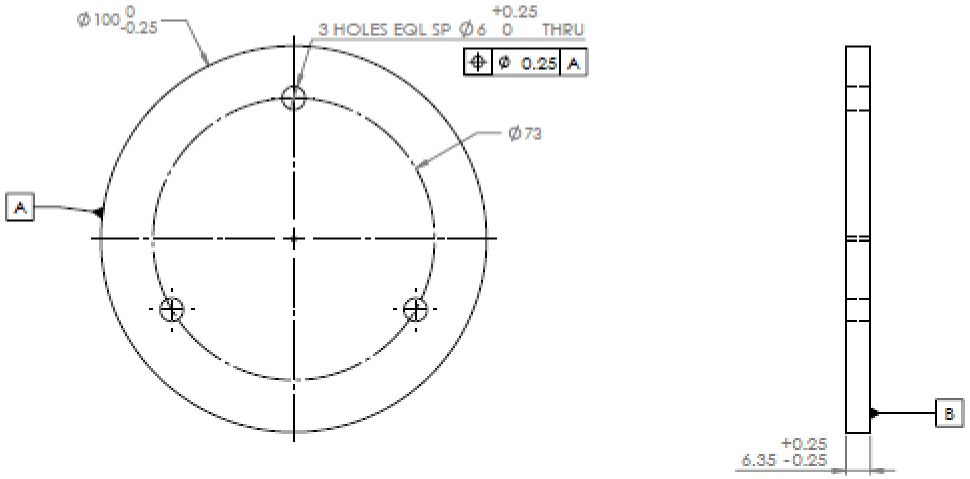


**Images of hand calculations available within the excel sheet!**

# Design

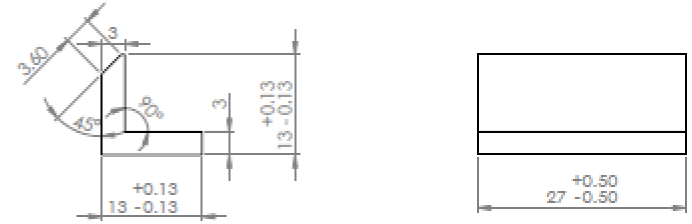

Threaded Fastener
Laser Mount
Brass Balls
(fastened to the top plate)
Acrylic Top and Bottom Plates
Brass V-Grooves

# Drawings

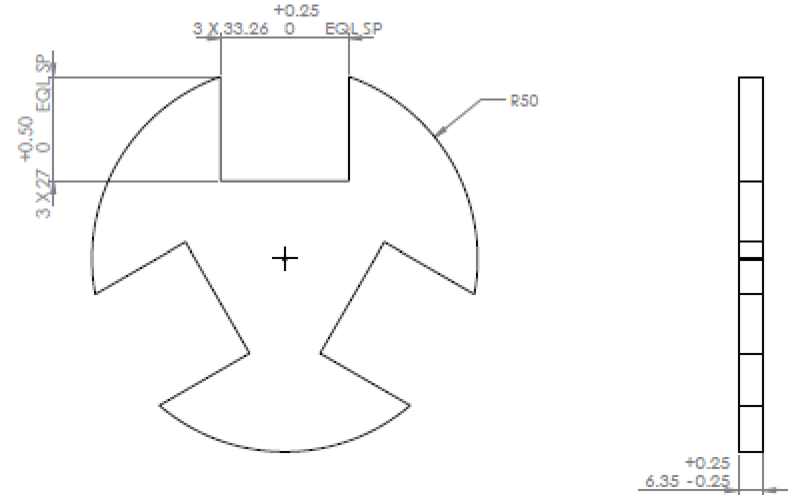



*Manufacturing Drawing of the Top Plate*

**Full scale drawings in Dropbox folder too!**

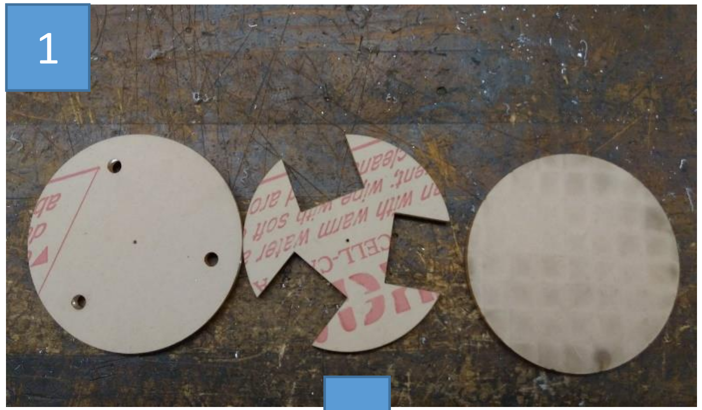



*Manufacturing Drawing of the Grooves*




*Manufacturing Drawing of the Slotted Plate*

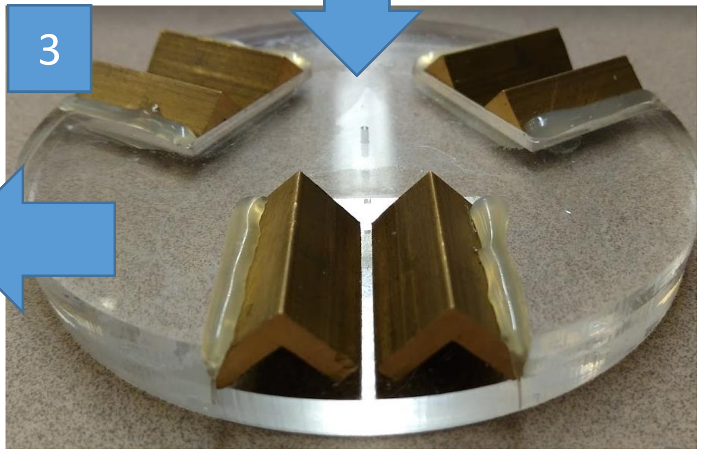
# Manufacturing




**Laser cutting the Plates** – 0.25 in thick acrylic

Taper observed but did not affect the functionality of KC

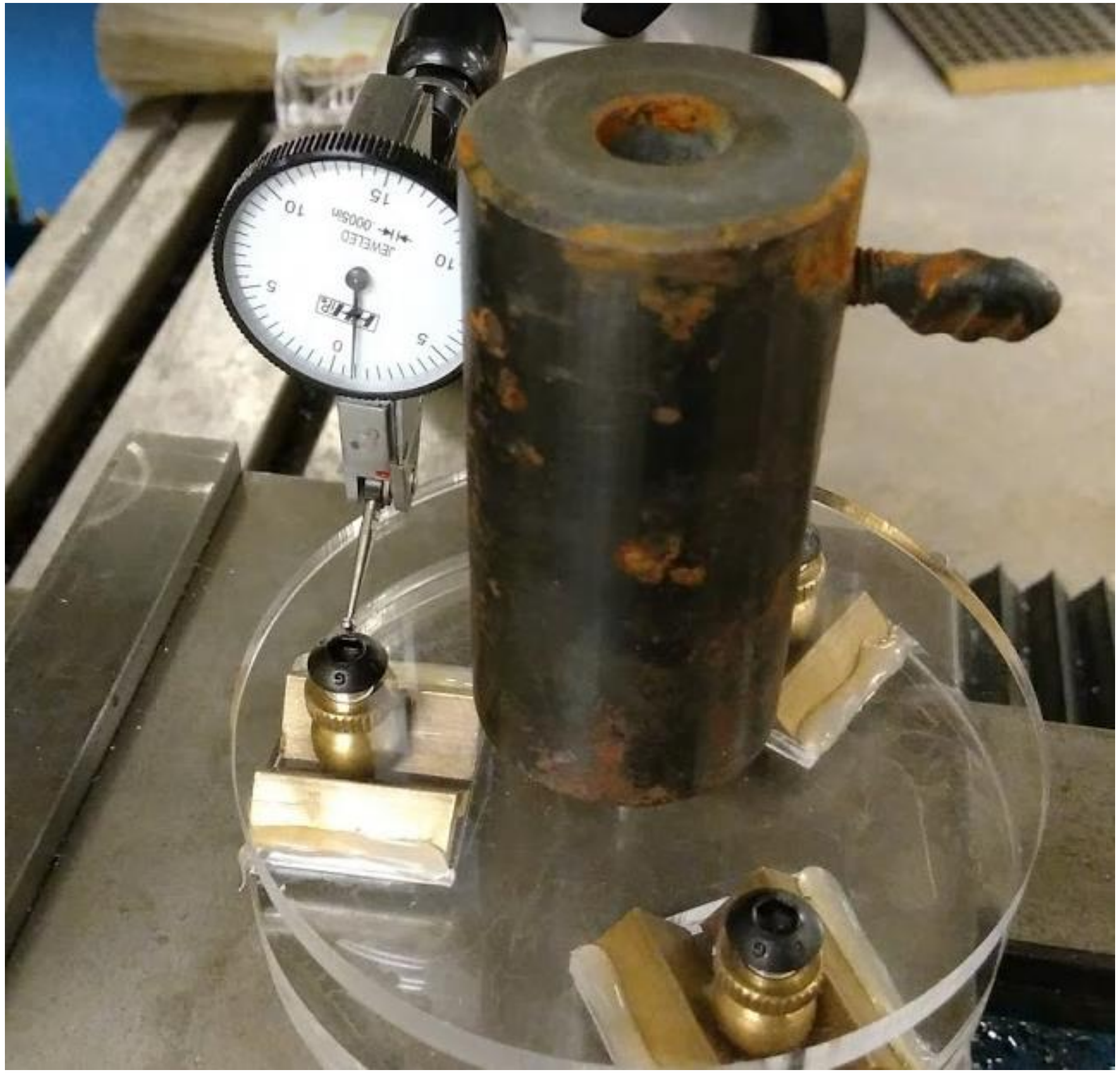

**Chamfering the grooves**– Brass – used a 45 deg chamfer tool




**Assembly** – Acrylic plates superglued

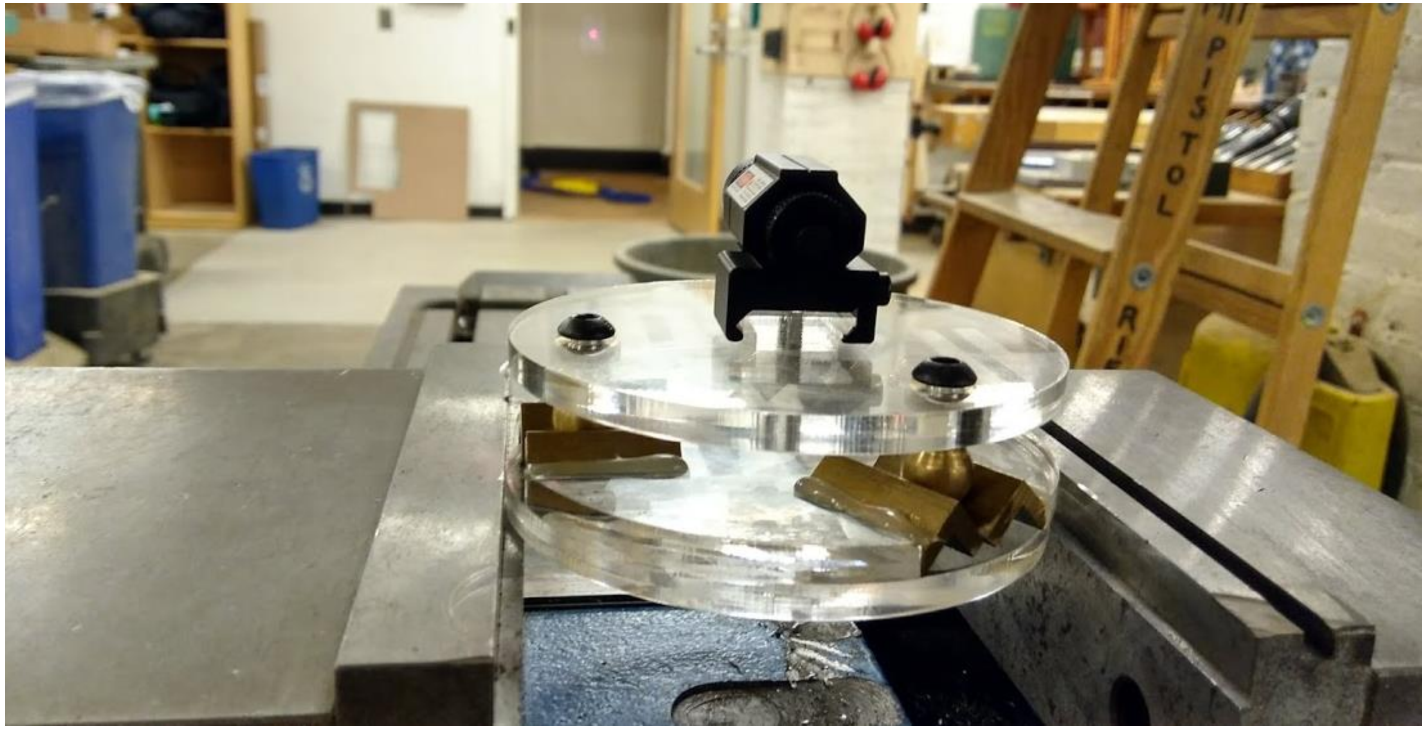
Grooves – potted with epoxy on the plate

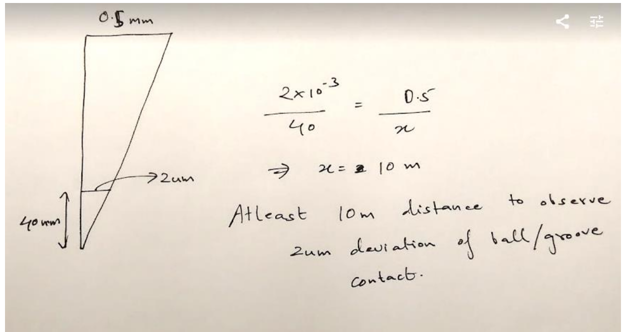
# Testing – Axial Stiffness



- A load of 1.26 Kg was applied at the center of the top plate of the coupling.
- The dial indicator was zeroed while the coupling was still loaded
- The load was removed and the change in the dial reading was noted.
- The deflection was measured as close as possible to the balls.
- The acrylic top plate deflected substantially causing a change in dial reading.
- I expected no change in reading because the predicted deflection of 0.4 um cannot be measured with conventional instruments.
- However, a single division change in the reading was observed (1 division = 0.0005 in or 12.7 um)
- This could possibly be due to the top plate deflection or slight movement of the grooves farther away from each other.

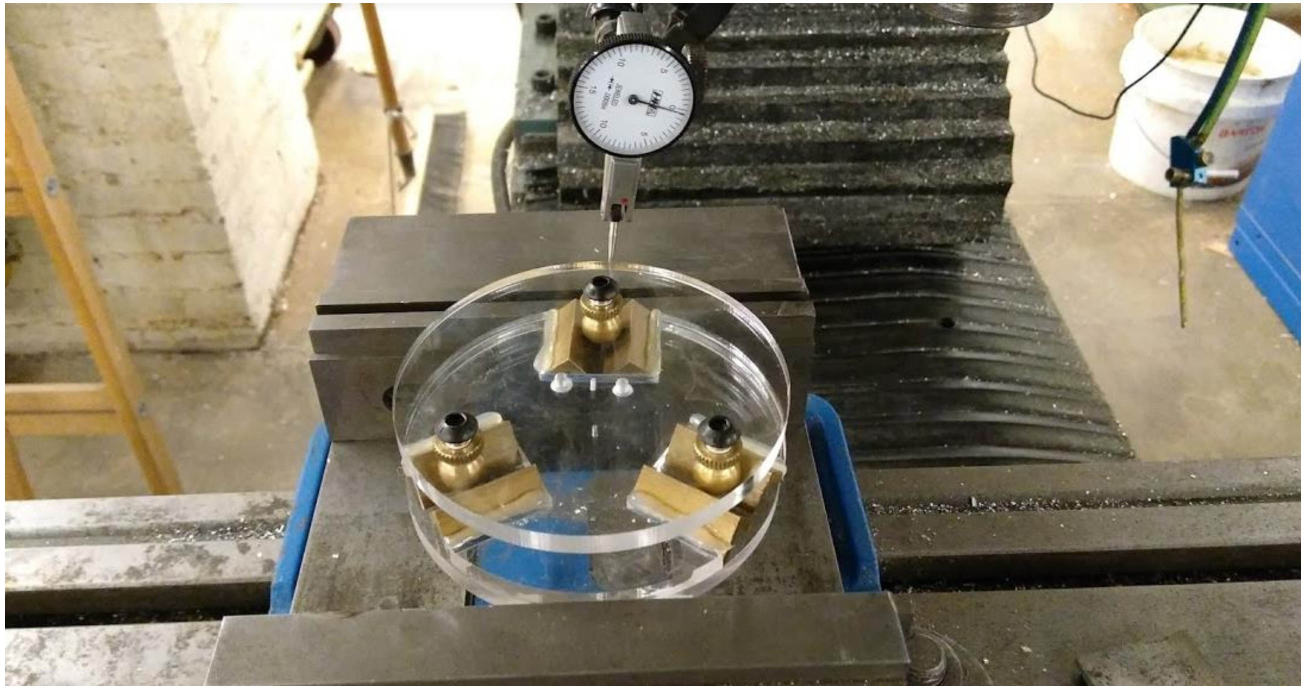
# Testing - Angular Repeatability




- The linear repeatability of a KC is estimated to be 2 um [1]
- Assuming a deviation of 2 um one of the ball/groove contacts, the minimum distance required to see a perceptible change (0.5 mm) in the laser spot is 10 m
- I did not observe any deviation of the laser spot 16 m far away even after 10 trials.
- Using simple geometry calculations, this suggests that the angular repeatability of the **KC is better than 31.2 micro-radians.**

# Testing – Linear Repeatability



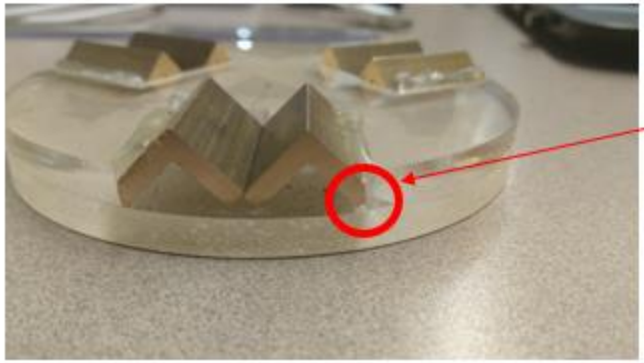
- To test the linear repeatability of the KC, I mounted the KC on the mill and tried to measure any deviation of the dial after each cycle (of lifting the top plate and putting it back)
- **As expected, the linear repeatability of the KC was better than what could be measured.**
- **The repeatability was at least better than 12.7 um**

# Peer Review Feedback and Improvements

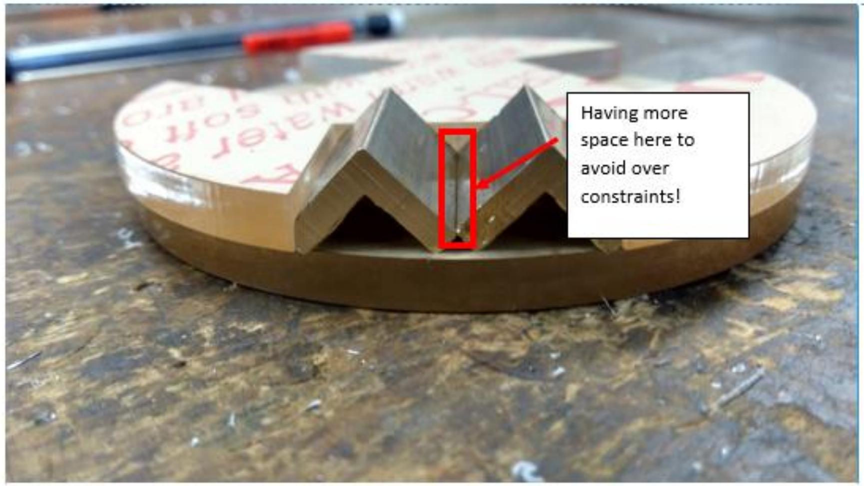
| Sr. No | Peer Feedback/Scope for Improvement | Taken care of? |
|---|---|---|
| | **Hardware/Test Improvements** | |
| 1 | Fixing the manufacturing errors in the kinematic coupling and resolving overconstraints | Yes (shown in next slide) |
| 2 | Proper Test Setup for Axial Stiffness measurement – ensure plate does not deflect | Yes |
| 3 | Proper Repeatability (Angular) test with good laser mount | Yes |
| | **Documentation Improvements** | |
| 1 | Reduce Significant Digits in design spreadsheets, Follow drawing conventions | Yes |

# Resolving overconstraints and errors in old KC

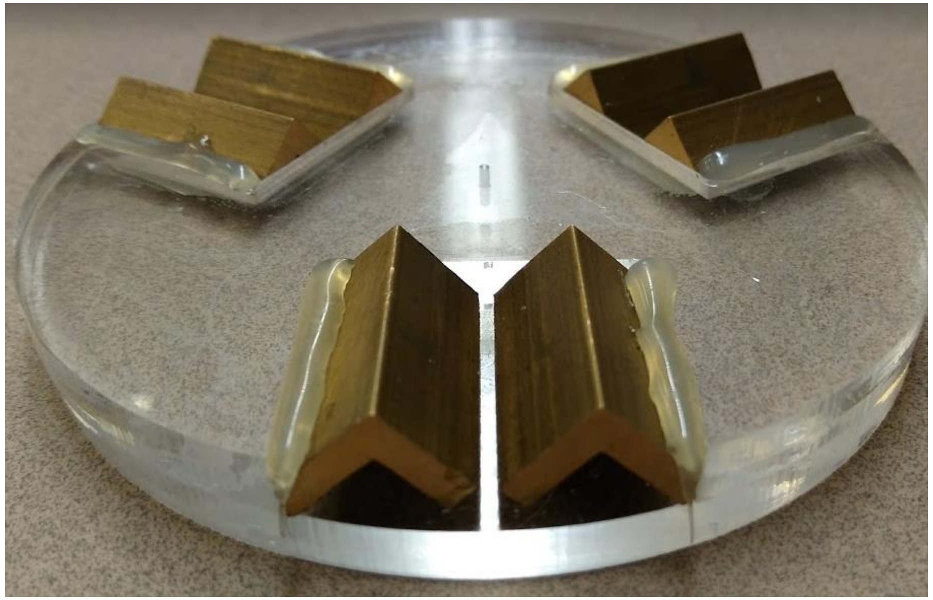
Old KC



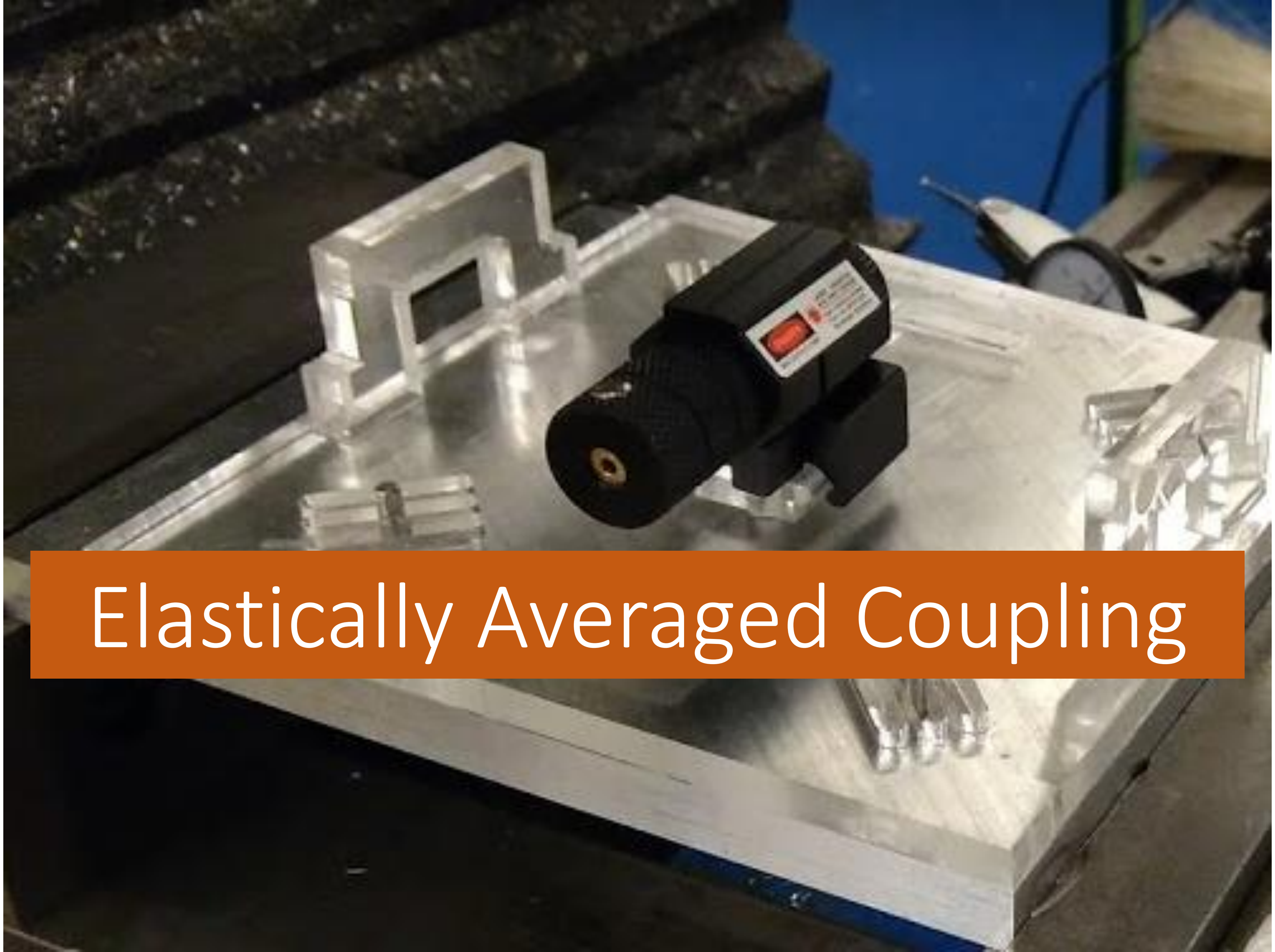
Not touching the base completely




Modified KC



# Elastically Averaged Coupling

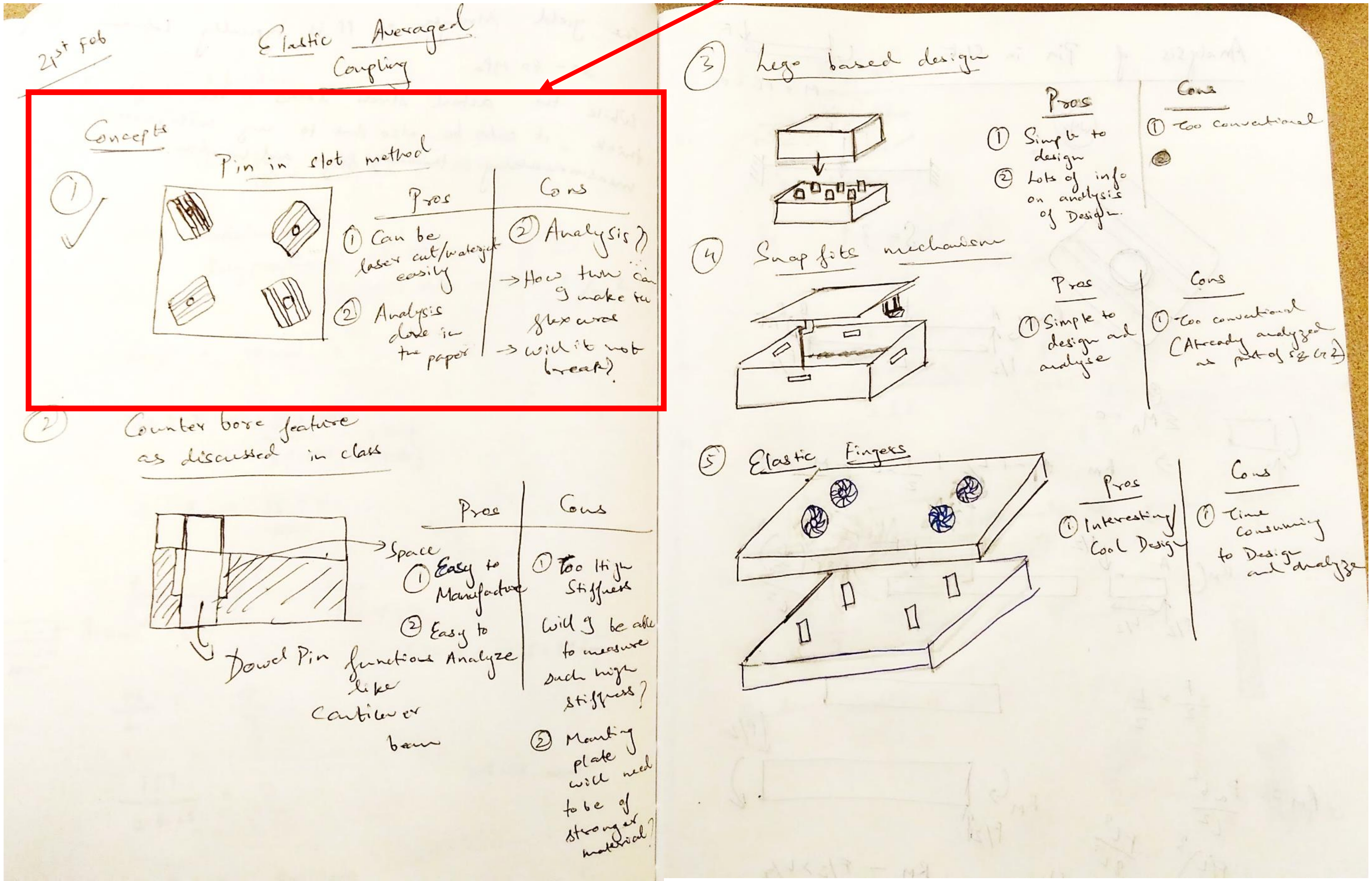


# FRDPPARC

| Sr. No. | Functional Requirements | Design Parameters | Analysis | References | Risks | Counter Measures |
|---|---|---|---|---|---|---|
| 1 | Ability to bear reasonable moment loads without failure | Yield Strength of the mounting plate<br>Moment of Inertia of the beam elements<br>Thickness of the beams | Bending stress calculations for the beam elements | Mechanics of Materials Textbook | The elastic beams may fail under excessive moment loads | Limit the amount of allowable moments or design a stop which does not allow the moments to exceed a limit |
| 2 | Accuracy | Manufacturing errors<br>Number of elastic contacts needed to average out the errors | Performance Ratio (PR) = Manufacturing error/desired accuracy<br>Number of elastic contacts = sqrt(PR) | A.H. Slocum, T.J. Teo, “Principle of elastic averaging for rapid precision design” | Too few elastic contacts for desired accuracy | Add more contact points |
| 3 | Moment Stiffness | V, W, Slot width, beam element properties, material properties, angle of orientation | Young's Modulus of the materials<br><br>Slot width, Dowel Pin diameter, thickness, width and length of the beam elements | Paper, "Principle of elastic averaging for Rapid Precision Design"<br><br>Mechanics of Materials Textbook | Too high stiffness which I cannot measure | Deterministic Design of the beam elements to get the stiffness I want |
| 4 | Ease of Manufacture | What do I have?<br>Time/Budget Constraints<br>Accessibility of Machines | Resource Assessment<br>Bank Balance<br>Machine Shop Schedules | MW Website, Hobby Shop site | I may not finish the EAC by the deadline. --Bad | Work fast and use readily available materials |
| 5 | Easy to insert and remove | Pull out force<br>Friction coefficient<br>Normal forces on the pin | Force on each beam<br>Pull out force calculation | Prior knowledge on mechanics of materials | The EAC is very difficult to insert and remove.<br>Hard to take repeatability readings | Design the beam elements such that the pull out force is nominal |

# Concept Generation

Selected



# Analysis

## 1 How many contact points needed to get the desired accuracy?

Manufacturing error = 200 um (Average Tolerance of the Mill – Kalpakjian)

Desired Accuracy = 100 um

Number of contact points needed = $\left[\frac{Manufacturing\ Error}{Desired\ Accuracy}\right]^2 = \boldsymbol{4\ contact\ points}$

## 2 Check for local yielding of elastic elements

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Elastic Averaging Spreadsheet - Check for yielding of the eleastic beam elements** | | | | | | Acrylic | |
| | | | | | Yield Strength of the Material | 70 | Mpa |
| Pin diameter | 3.2 | mm | | | Tensile Modulus | 3102 | Mpa |
| Slot diameter | 3 | mm | | | Flexural Modulus | 3100 | Mpa |
| Displacement of beam element due to Dowel Pin offset | 0.1 | mm | | | | | |
| Total deflection to be borne by a flexure | 0.2 | mm | | | | | |
| Length of the flexure | 20 | mm | | | | | |
| Thickness of the sheet | 6.2 | mm | | | | | |
| Width of the flexure | 1.65 | mm | | | | | |
| Moment of Inertia | 2.3 | mm4 | | | | | |
| Stiffness of each flexure | 172.8 | N/mm | | | | | |
| Force on each beam | 34.6 | N | | | | | |
| Bending Stress | 30.7 | N/mm2 | | | | | |
| Bending Stress Ratio | 0.44 | | | | | | |

# Analysis

## 3 Estimating the System Stiffness

```
% 2.77 Precision Machine Design 2018
% Akshay Harlalka - 23rd Feb, 2018
%Reference for Calculations:
%% EAC Calculations

% Defining the number of elastic contacts needed
% deltaa=input('Enter the desired accuracy of the system in um')
% deltam=input('Enter the expected manufacturing error in the parts in
% perratio=deltam/deltaa;
% n=ceil(sqrt(perratio))%
thetai=[pi/4 (pi/4+pi/2) (pi/4+pi) ((pi/4)+(3*pi/2))]

%Geometry
V=2.3*10^-3
W=0.675*10^-3
r=0.05176 % coupling radius
%Orientation Matrix
G1=[cos(thetai(1)) cos(thetai(1)+pi/2) 0 ; sin(thetai(1)) sin(thetai(1)+pi/2
G2=[cos(thetai(2)) cos(thetai(2)+pi/2) 0 ; sin(thetai(2)) sin(thetai(2)+pi/2) 0 ; 0 0 1]
G3=[cos(thetai(3)) cos(thetai(3)+pi/2) 0 ; sin(thetai(3)) sin(thetai(3)+pi/2) 0 ; 0 0 1]
G4=[cos(thetai(4)) cos(thetai(4)+pi/2) 0 ; sin(thetai(4)) sin(thetai(4)+pi/2) 0 ; 0 0 1]
%Jacobian
Ji1=[1 0 0 ; 0 1 r ; 0 0 1]
Ji2=[1 0 0 ; 0 1 r ; 0 0 1]
```

**Key Highlights**

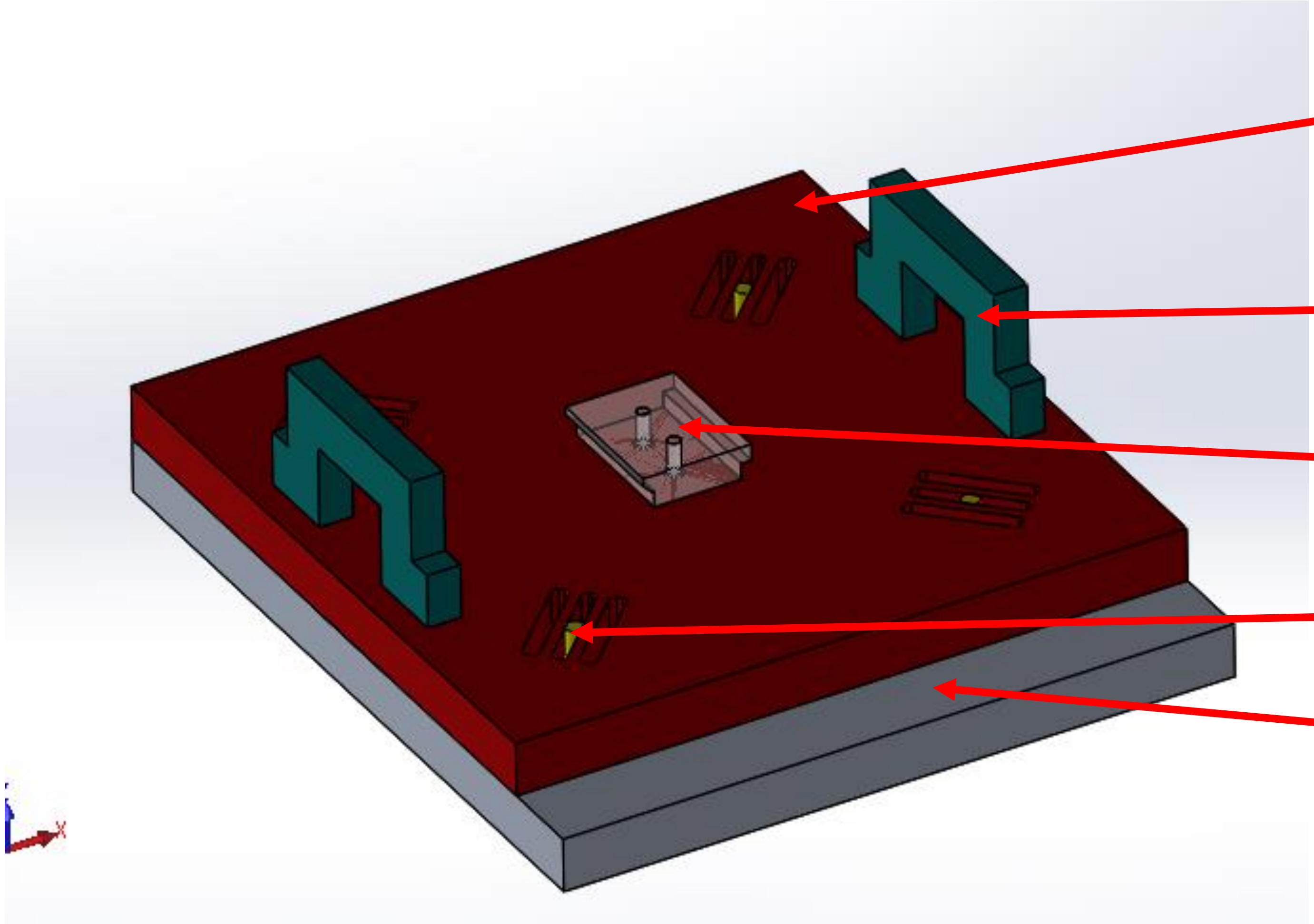
Ignores the effect of body stiffness. Assumes body has infinite stiffness

Uses the calculation strategy outlined in the paper titled, "Principle of elastic averaging for rapid precision design" [2]

Final System Stiffness Predicted = 3730 Nm

Code is uploaded in the Dropbox folder too!

# Design



Mounting plate

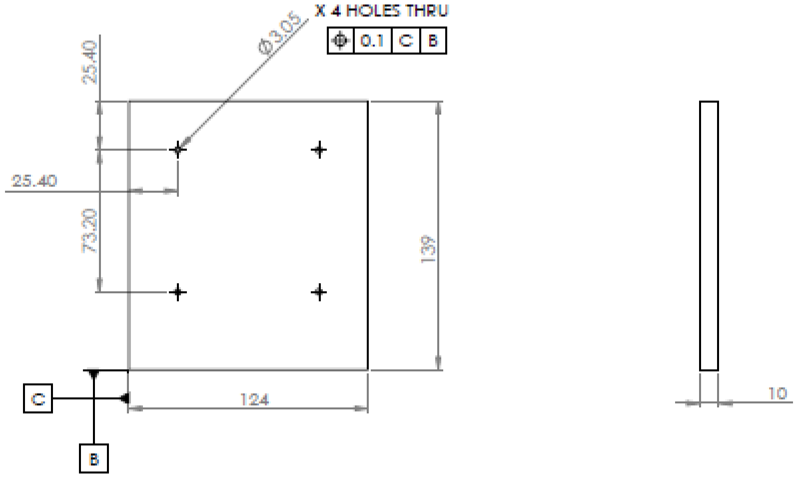
Handle for easy insertion and removal of the mounting plate

Mount for the laser

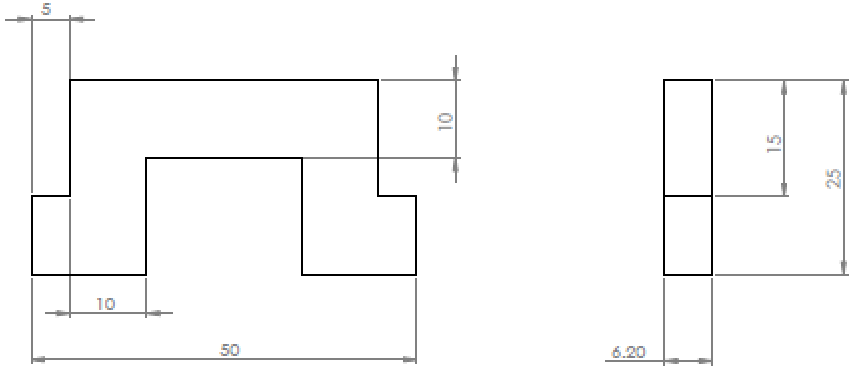
Dowel Pins

Base plate (Extra space for clamping)

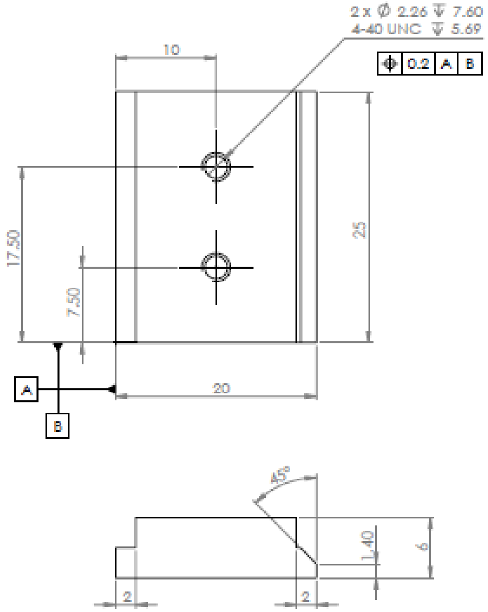
# Drawings




**Base Plate**




**Holder**

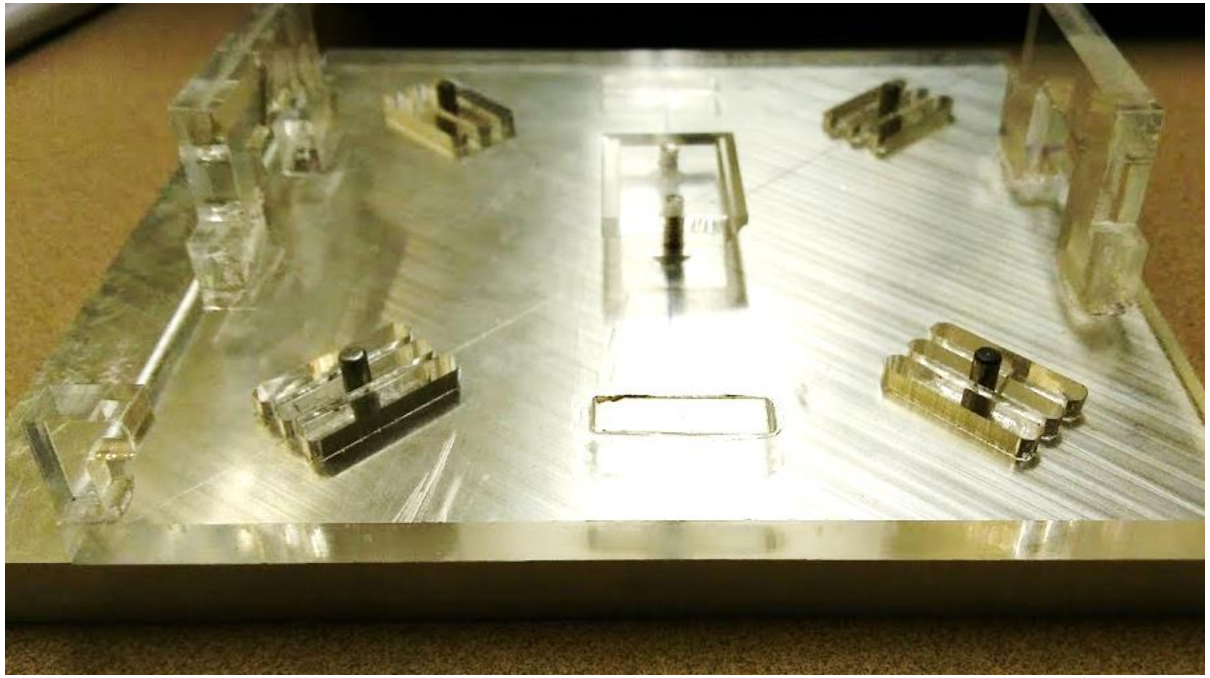



**Laser Mount**

*Where tolerances are not assume +/- 0.1 mm. Mentioned in full-scale drawing files*

**Individual drawing files will full details are available in the folder. This is for representational purposes only.**

# Manufacturing



**Materials and Manufacturing :**

- **Mounting Plate, Laser Mount, Holder** – All fabricated by laser cutting Acrylic 6.2 mm thick sheet
- **Base Plate** – Aluminium 10 mm thick plate – Took original stock size = 124 * 224 * 10 mm – Drilled 4 holes using 0.12 inch drill, followed by 1/24 ‘’ inch reamer (through) and 1/26 inch reamer (upto 1mm depth) to allow the pins to stand straight while press-fitting dowel pins to plate
- **Dowel Pins** – Stainless Steel – Available off the shelf

# Taper due to Laser cutting

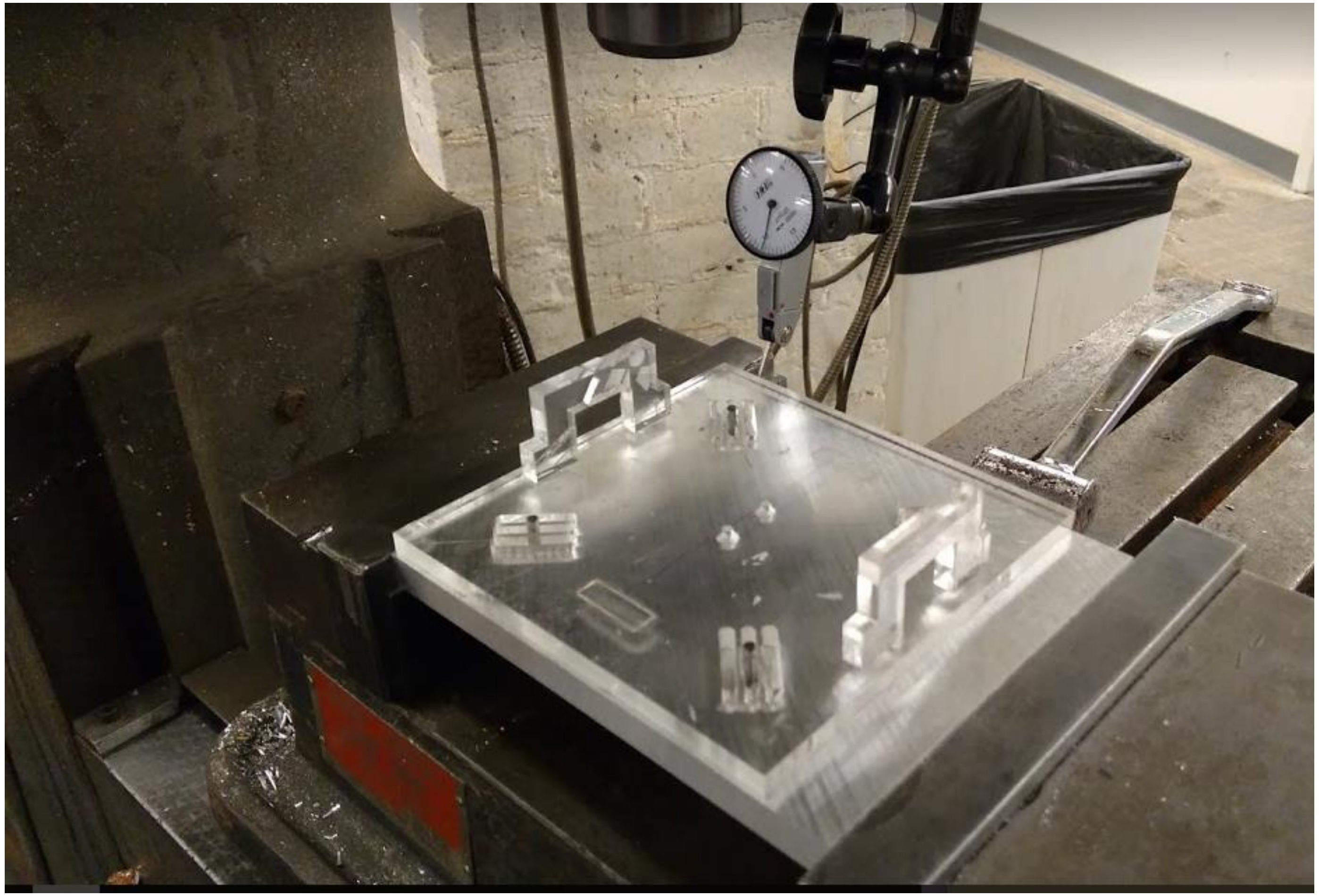
**Taper Calculation**

- Laser cutting process introduces taper through the thickness of the part being cut
- Experimentation with various laser power/speed settings was done to minimize the taper. However, taper could still not be eliminated.
- The width of the slots on the top and the bottom plate were both measured and the taper angle calculation was done. A 0.2 mm difference was found between the widths. Thickness of sheet was 6 mm
- This meant that the taper angle was 1.7 degrees !

**Effect of Taper and refined performance expectations from the coupling**

- Based on the introduction of taper, I expected the stiffness of my coupling to be lower than predicted. Also, I expected that taper will help accelerate the process of wear and cause the coupling to be less repeatable.

# Testing - Accuracy



- In last week's report, the mounting plate was laser-cut while the bottom plate was machined. I wasn't able to measure the accuracy of the EAC last week as the manufacturing errors were different in both processes.
- This week, I machined two adjacent edges of the top and bottom plate separately, on the same machine and after assembling the coupling, measured the deflection of the X and Y edges using a dial indicator.
- While I tried my best to machine the sides of the acrylic plate and aluminium as with the same accuracy, it seemed there was an offset each time. It could be due to the fact that Acrylic is much less stiffer than aluminium and the cutting forces at 1500 rpm were deflecting it substantially.
- The results are shown in the next slide.

# Testing - Accuracy

**1 division = 0.0005 in = 12.7 um**

| | | X Axis | | | | Y Axis | | | |
|---|---|---|---|---|---|---|---|---|---|
| Sr. No | Distance (mm) | Dial Reading of Bottom Plate | Dial Reading on Top Plate | Difference | Difference after subtracting offset | Dial Reading of Bottom Plate | Dial Reading on Top Plate | Difference | Difference after subtracting offset |
| 1 | 0 | 22 | 12 | 10 | **10** | 0 | 22 | 22 | **11** |
| 2 | 10 | 19 | 10 | 9 | **9** | 0 | 19 | 19 | **8** |
| 3 | 20 | 19 | 12 | 7 | **7** | 0 | 17 | 17 | **6** |
| 4 | 30 | 18 | 10 | 8 | **8** | 0 | 16 | 16 | **5** |
| 5 | 40 | 15 | 9 | 6 | **6** | 0 | 15 | 15 | **4** |
| 6 | 50 | 12 | 8 | 4 | **4** | 0 | 13 | 13 | **2** |
| 7 | 60 | 10 | 5 | 5 | **5** | 0 | 13 | 13 | **2** |
| 8 | 70 | 7 | 4 | 3 | **3** | 0 | 12 | 12 | **1** |
| 9 | 80 | 5 | 2 | 3 | **3** | -1 | 9 | 11 | **0** |
| 10 | 90 | 3 | 2 | 1 | **1** | -2 | 9 | 11 | **0** |
| 11 | 100 | 1 | 1 | 0 | **0** | -2 | 9 | 11 | **0** |
| 12 | 110 | 0 | 0 | 0 | **0** | -2 | 9 | 11 | **0** |

# Testing - Accuracy

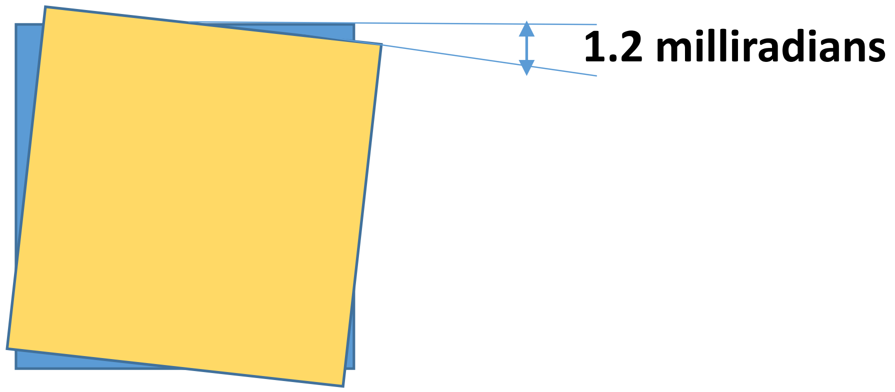
| | | X Axis | Y Axis |
|---|---|---|---|
| Sr. No | Distance (mm) | Deviation (um) | Deviation (um) |
| 1 | 0 | 127 | 140 |
| 2 | 10 | 114 | 102 |
| 3 | 20 | 89 | 76 |
| 4 | 30 | 102 | 64 |
| 5 | 40 | 76 | 51 |
| 6 | 50 | 51 | 25 |
| 7 | 60 | 64 | 25 |
| 8 | 70 | 38 | 13 |
| 9 | 80 | 38 | 0 |
| 10 | 90 | 13 | 0 |
| 11 | 100 | 0 | 0 |
| 12 | 110 | 0 | 0 |

The maximum deviation observed in the coupling assembly was **140 um** which was better than the accuracy of the mill used to machine its components!

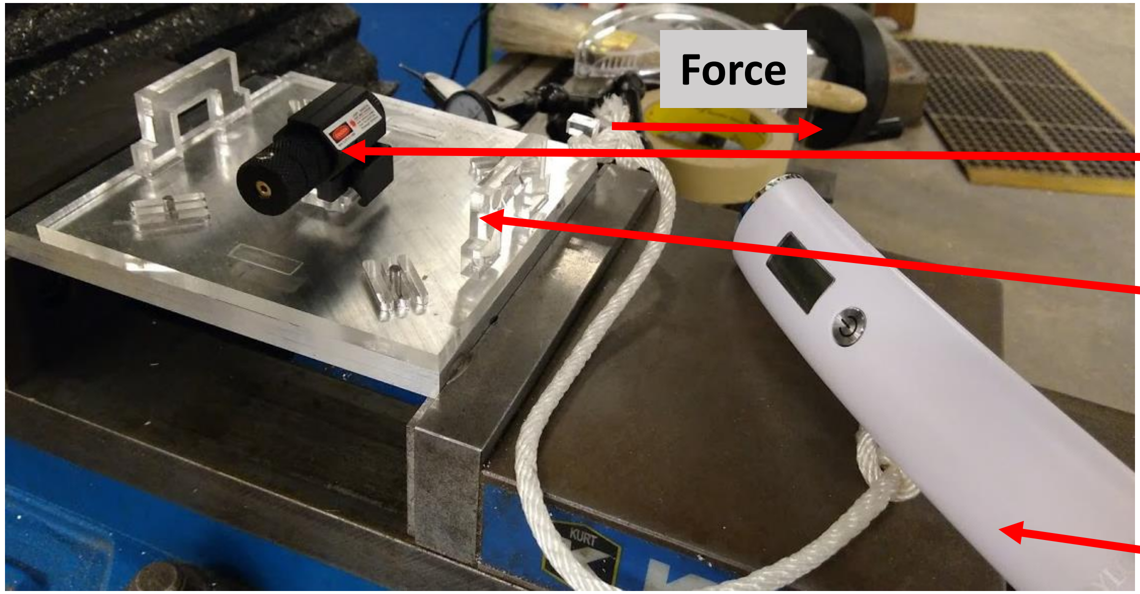
This shows us the power of elastic averaging!

**Expected accuracy was around 100 um**

Angular deviation was **1.2 milliradians**



# Testing – Moment Stiffness about Z axis




Laser was fixtured firmly on the mount

The handle were attached to a laser-cut acrylic plate using acrylic solvent while the laser mount was attached using threaded fastener

Digital scale for load measurement

**Important Note:** I did not have access to a digital torque wrench. So, I applied a single force to cause a moment instead of a force pair. This **will cause translational motion of the plate** as well. However, the translational motion of the plate will not get amplified whereas the angular deviation will. Therefore, if the distance from the laser is very large, the deviation of the laser due to translational motion **can be neglected** in comparison to the angular deviation .

# Testing Results - Stiffness

**Predicted Stiffness was 3730 Nm**

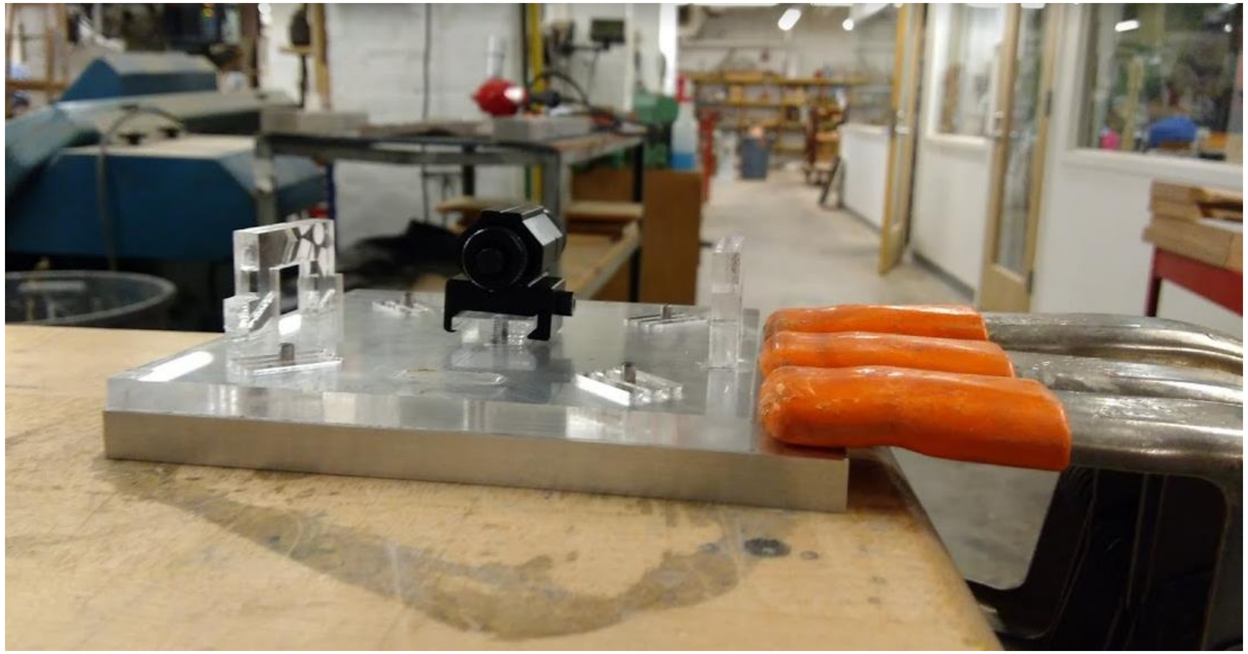
| Sr. No | Weight (in Kg) | Deflection on paper (mm) | Distance (m) | Theta (milli-radians) | Force (N) | Arm length (mm) | Moment (Nm) | Stiffness (Nm) |
|---|---|---|---|---|---|---|---|---|
| 1 | 2.5 | 5.74 | 8 | 0.72 | 25 | 62 | 1.5 | 2152 |
| 2 | 3.5 | 10.72 | 14 | 0.77 | 34.3 | 62 | 2.1 | 2777 |
| 3 | 4.1 | 10.10 | 14 | 0.72 | 40.9 | 62 | 2.5 | 3512 |
| 6 | 4.5 | 8.80 | 8 | 1.1 | 44 | 62 | 2.7 | 2454 |
| 4 | 4.8 | 14.86 | 14 | 1.06 | 47 | 62 | 2.9 | 2748 |
| 5 | 4.8 | 10.99 | 8 | 1.38 | 47 | 62 | 2.9 | 2109 |

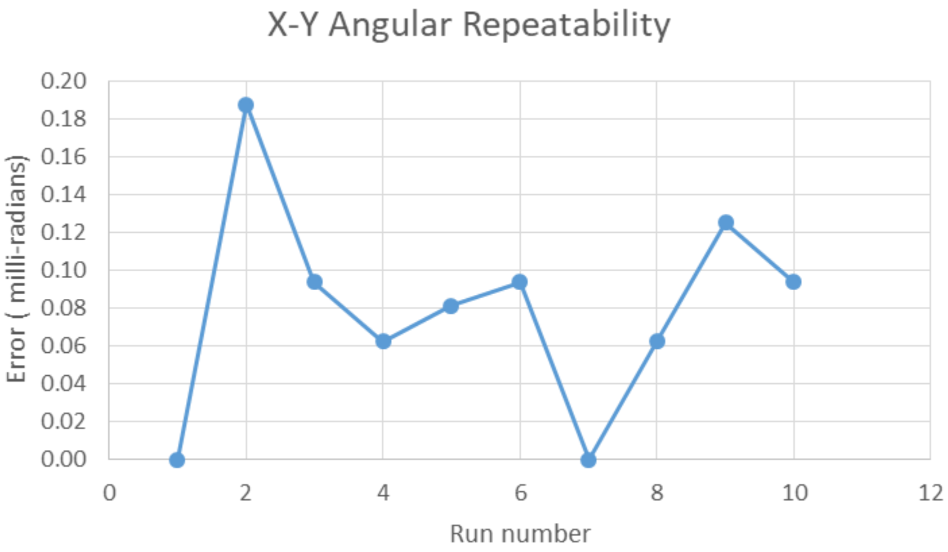
Values reported last week    Recent Values

**Probable reasons for difference in the theoretical and experimental Stiffness Values:**

1. Theoretical Stiffness does not take the body stiffness into consideration. Therefore, actual stiffness will be slightly lower than predicted.
2. Taper due to the laser-cutting process will reduce stiffness.
3. Coupling getting less stiff over time. Multiple repeatability tests were conducted during this time. Possibility that wear of the acrylic beams is causing the reduction in stiffness.

# Testing – Angular Repeatability



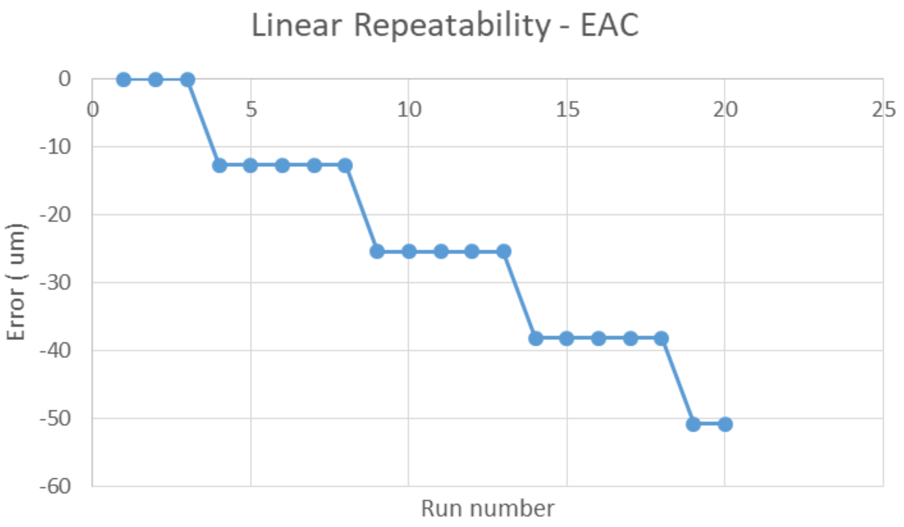


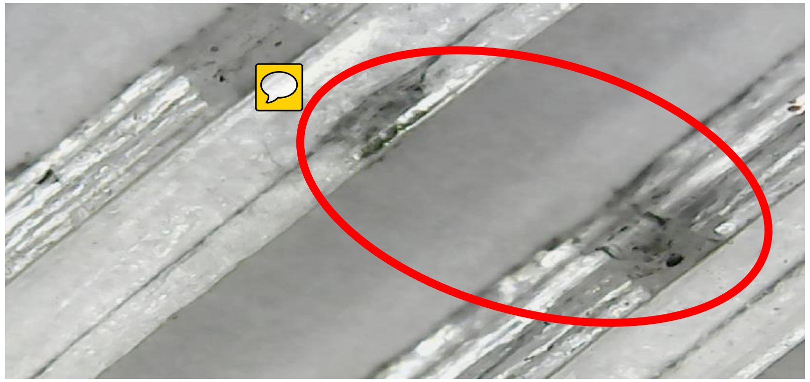

- To determine the angular repeatability, the base plate was clamped using 3 spring clamps. Unfortunately, I did not leave enough space for clamps on the other side, so had to clamp it from one side only.

- The target was 16 m away. 10 repeatability measurements were taken. The repeatability graph is shown above

- This time, laser was mounted firmly. So, that uncertainty was eliminated this week.

# Testing – Linear Repeatability



40 X Microscopic Image of dowel pin/elastic beam contact

- The repeatability test for EAC showed weird results.
- The repeatability seemed to be worsening over time
- One possible explanation could be the wear of acrylic beams by dowel pins.
- Evidence of wear could be observed in the microscopic image

# Peer Review Feedback and Improvements

| Sr. No | Peer Feedback/Scope for Improvement | Taken Care of? |
|---|---|---|
| | **Hardware/Test Improvements** | |
| 1 | Angular repeatability test for EAC need to be redone – as laser was not fixtured properly. | Yes |
| 2 | EAC's are mainly to improve accuracy – therefore important to measure accuracy to close the design loop | Yes |
| 3 | Moment Stiffness | Yes |
| | **Documentation Improvements** | |
| 1 | Elastic Averaging formula used was wrong – typo on the paper | Yes |
| 2 | Reduce Significant Digits in design spreadsheets, Follow drawing conventions | Yes |

# References

[1] Martin L. Culpepper, Carlos Araque and Marcos Rodriguez, “Design Of Accurate And Repeatable Kinematic Couplings” , Web Article http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.510.4074&rep=rep1&type=pdf (Accessed on 2nd March, 2018)