Week 9

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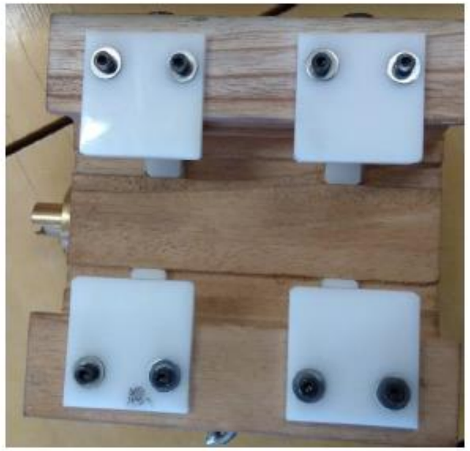
Work completed at the end of Week 7

- Both LM Modules Manufactured.
- Testing of the modules completed
- Error Model (Geometric and Load Induced) implemented

Manufacturing



Backside of the carriage

Both Modules Manufactured

Work done in Week 9

- Selection of the leadscrew – Analysis including shaft whip and bending and buckling
- Selection of bearings – using the radial and thrust forces
- Sketch Model of the leadscrew Mounting
- SolidWorks Model of the T-Base Assembly (Rough)
- Geometric Error Model of the complete Machine.
- Seek and Geek 9

Leadscrew Selection - Analysis

Buckling Analysis

Buckling Analysis

The Buckling analysis is done for a fixed-fixed condition.

Critical Buckling Load $(P_{cr}) = \dfrac{4\pi^2 EI}{L^2}$

$$P_{cr} = \frac{4\pi^2 (200\times10^3)\frac{\pi}{64}(d^4)}{(115)^2}$$

$$d^4 = \frac{P_{cr}\,(115)^2 \times 64}{4\pi^2 (200\times10^3)\,\pi}$$

Now, the critical buckling load should atleast be equal to the force in thrust direction.

$\therefore \quad P_{cr} \geq 16.8\text{ N}$

Considering a FOS of 1.4.

$P_{cr} \geq 23.52.$

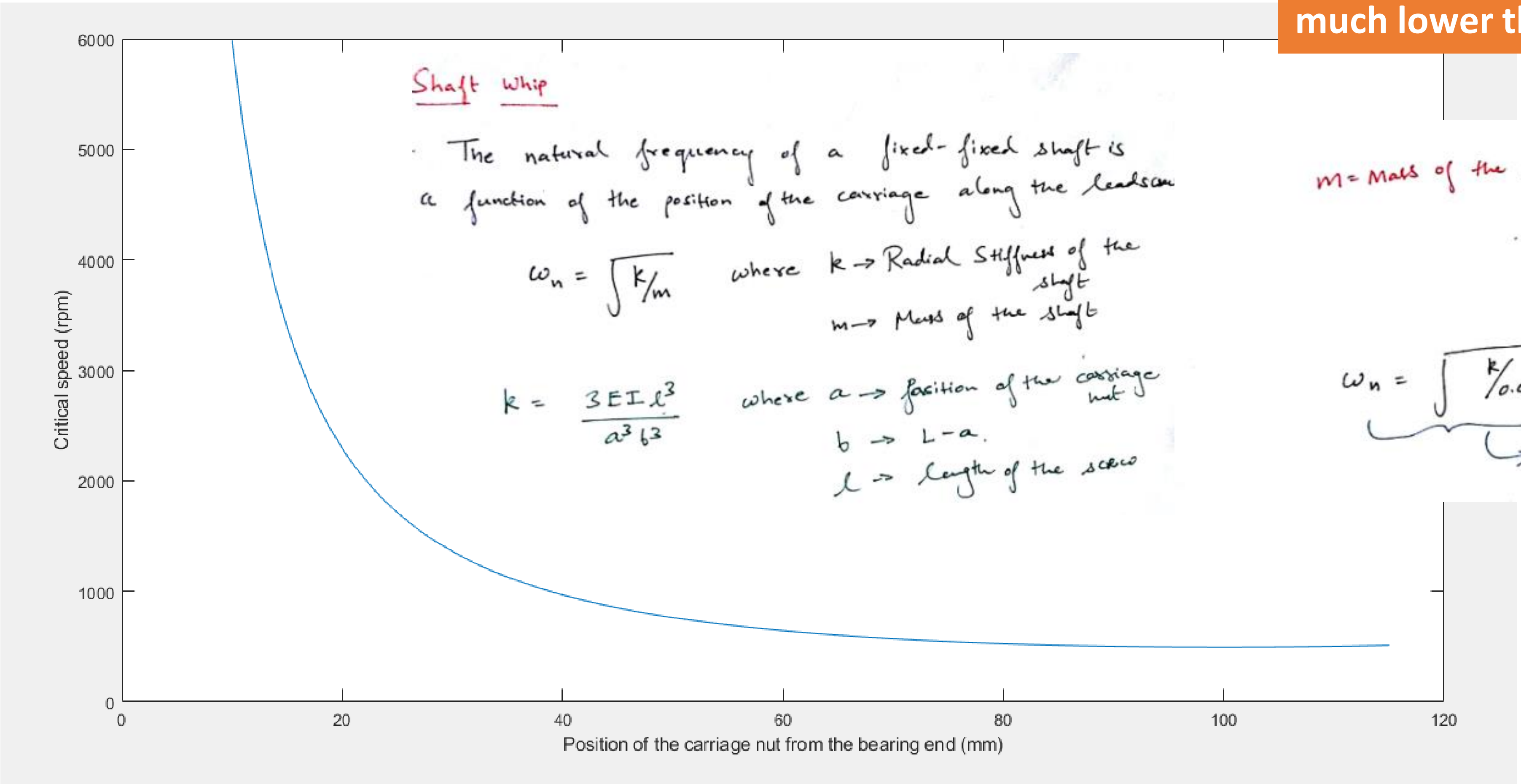
$$\therefore \quad d^4 = \frac{(23.52)(115)^2(64)}{4\pi^2 (200\times10^3)\,\pi}$$

$$\boxed{d = 0.946\text{ mm}}$$

Leadscrew Selection - Analysis

Shaft Whip Analysis

Lead screw needs to be operated below 950 rpm! Actual operation speeds will be much lower than this



Shaft Whip

The natural frequency of a fixed-fixed shaft is a function of the position of the carriage along the leadscrew

$\omega_n = \sqrt{k/m}$ where $k \rightarrow$ Radial Stiffness of the shaft

$m \rightarrow$ Mass of the shaft

$k = \frac{3EI\ell^3}{a^3 b^3}$ where $a \rightarrow$ position of the carriage nut

$b \rightarrow L - a$.

$\ell \rightarrow$ Length of the screw

m = Mass of the shaft = $\frac{\rho \pi d^2}{4} \ell$

$= \frac{8000\ kg/m^3 \cdot \pi (8\times10^{-3})^2 (0.2)}{4}$

= 80 grams

$\underbrace{\omega_n = \sqrt{k/0.08}}$ $k \rightarrow$ function of a.

Plotted in matlab!

Leadscrew Selection - Analysis

Misalignment Budget Analysis
(Test Case: Assuming Carriage at centre of leadscrew)

Shaft Bending Analysis

* What is the minimum force that can cause the yielding of shaft through bending?

For the fixed-fixed beam, the max deflection will be at the center of the leadscrew. Also, max bending moment is at the center!

$$M = \frac{F}{2}\cdot\frac{L}{2} = \frac{FL}{4}$$

$$y = \frac{d}{2}$$

$$I = \frac{\pi}{64}d^4$$

Actual force experienced by the leadscrew will be less as some of the misalignment will be absorbed by the wave spring in the carriage assembly

$$\sigma_b = \frac{My}{I} = \frac{\frac{FL}{4}\left(\frac{d}{2}\right)}{\left(\frac{\pi}{64}d^4\right)} = \frac{FL/8}{\frac{\pi}{64}d^3}$$

$$\sigma_b = \frac{8FL}{\pi d^3}$$

Yield stress for mild steel = 250 MPa.

$$250 = \frac{8FL}{\pi d^3}$$

$$250 = \frac{8F(200)}{\pi(5.5)^3}$$

$$F = \frac{(250)\pi(5.5)^3}{8(200)}$$

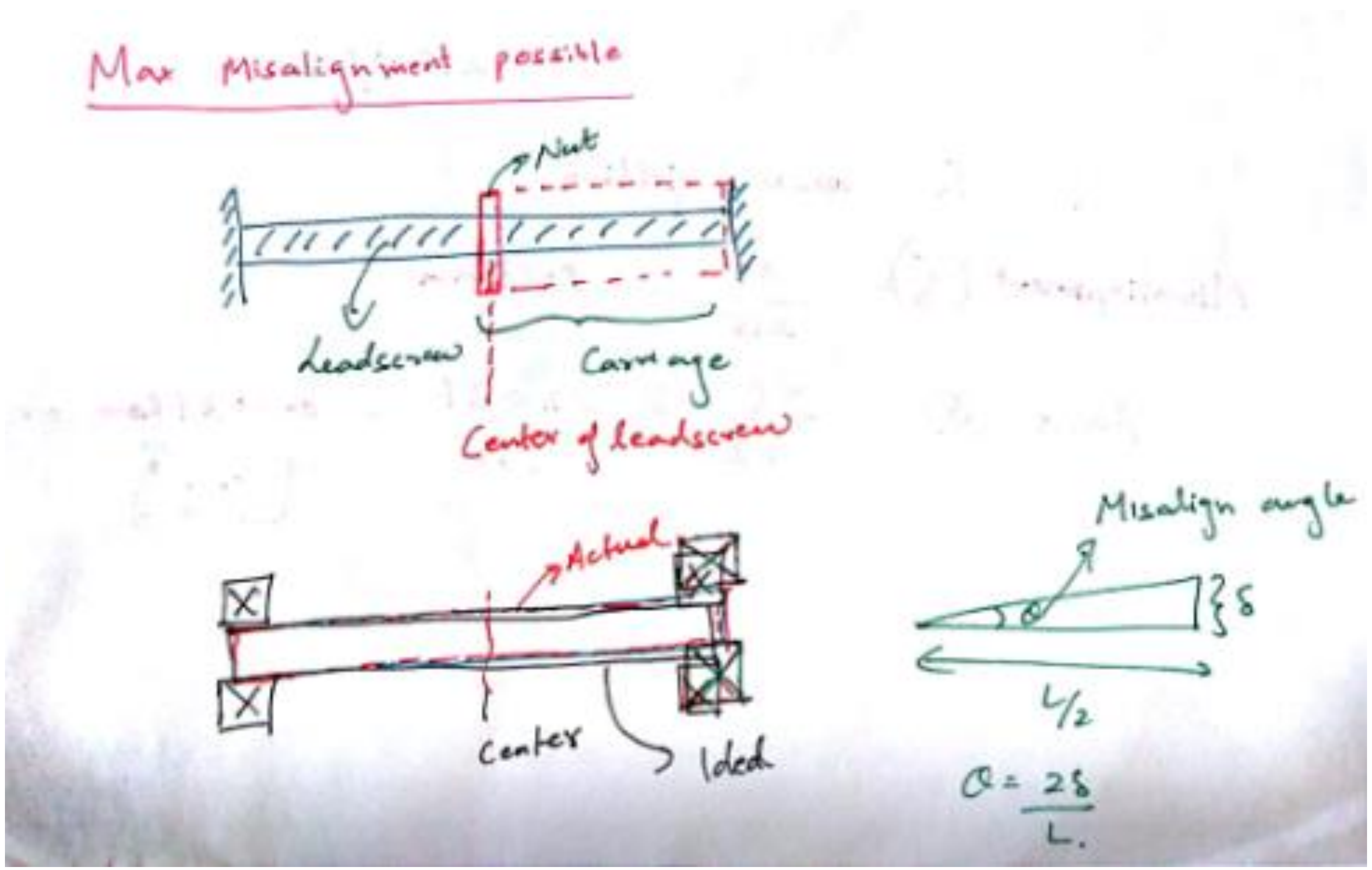
$$\boxed{F = 81\ N}$$

Therefore, 81 N is the minimum force required to cause the yielding of the beam / leadscrew

∴ The radial force due to misalignment needs to be less than this force.

Leadscrew Selection - Analysis

Misalignment Budget Analysis
(Test Case: Assuming Carriage at centre of leadscrew)



Radial Stiffness of Leadscrew

$$\delta = \frac{FL^3}{192EI} \quad \text{(At the center)}$$

$$k = F/\delta = \frac{192EI}{L^3}$$

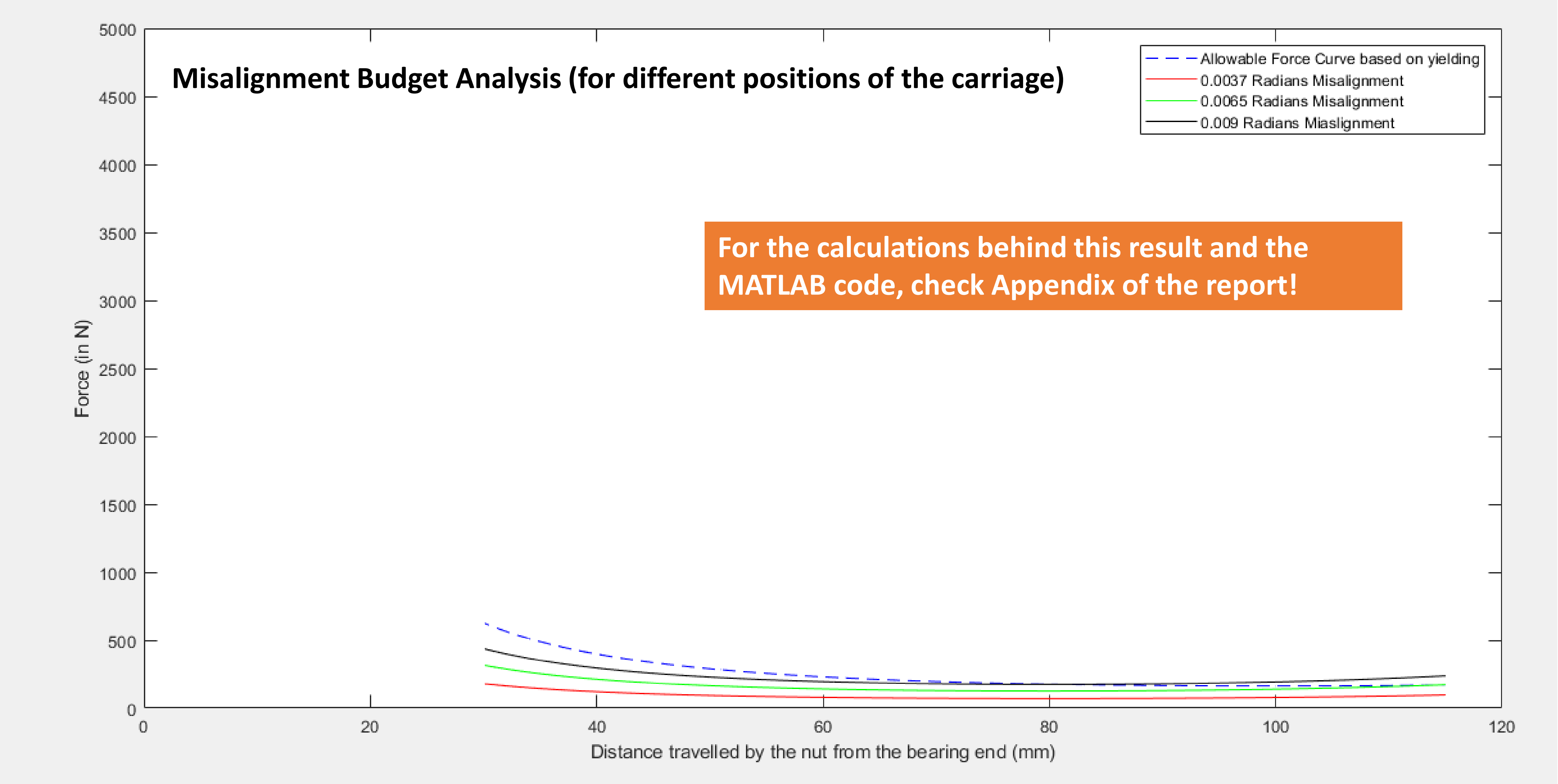
$$I = \frac{\pi}{64}d^4, \quad \therefore k = \frac{192\,E\left(\frac{\pi}{64}\cancel{E}d^4\right)}{L^3}$$

$$k = \frac{192E\pi d^4}{64L^3}$$

$$= \frac{192(200\times10^3)\pi(5.5)^{\cancel{3}4}}{64(200)^3}$$

$$= 215\ \text{N/mm}$$

Leadscrew Selection - Analysis



Leadscrew Selection

Data

Buckling Load = 26,821 N

Shaft Whip Frequency = Refer to graph (Function of carriage position)

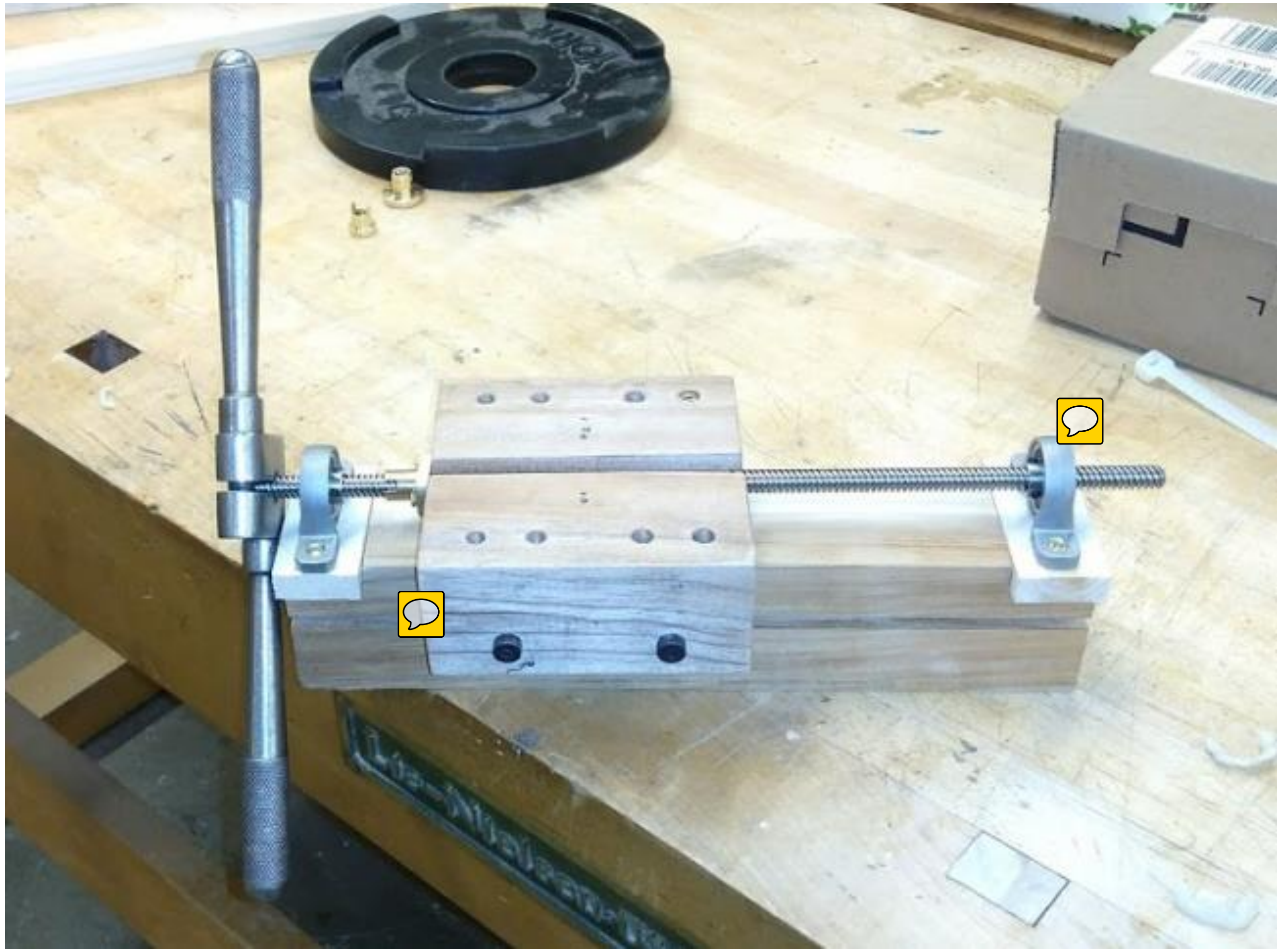
I already had a 8mm leadscrew with 2 mm lead. The antibacklash nut also came as part of the same set. So, I did not want to make changes to the design.

Following the analysis shown in the previous slides, it was clear that the selected leadscrew will meet the requirements.

Reasons why this leadscrew was ordered

Cheapest available
Also came with the anti-backlash assembly

Sketch Model and Link to Video



Link to video

Bearing Selection

The equivalent static load rating for the bearing is given by;

$$P_0 = X_0 F_r + Y_0 F_a$$

In this case $Fa/Fr < e$ and is close to zero as the axial load is 10 times lesser than the radial loads.

Assuming a factor of safety of 1.4,

Equivalent Radial Static Load = 1.4*200 =280 N

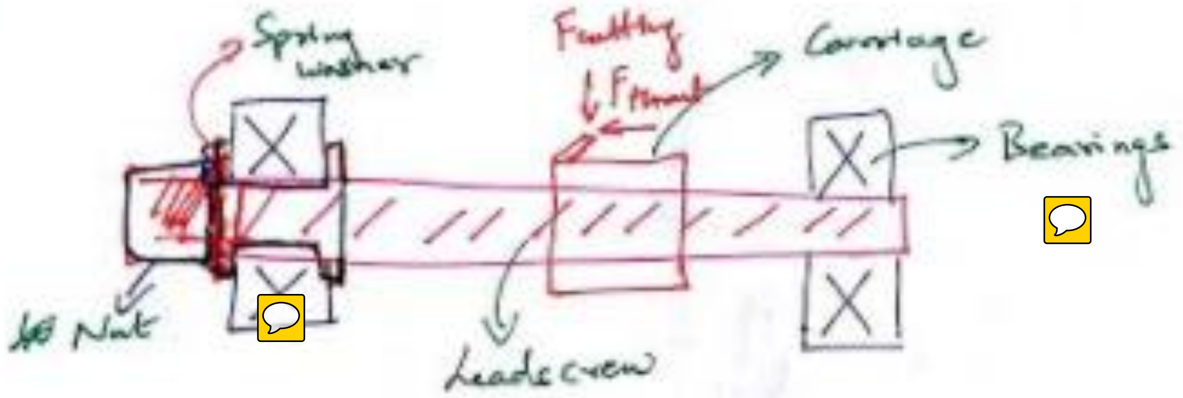
I needed the ID of the bearing to be 8 mm

For the 608RS bearing the Basic Static load capacity =1350 N
Dynamic load capacity= 3390 N

I already had 608 RS bearings in stock before hand so I wanted to check if these can be used directly

Therefore, the 608RS bearing can meet the requirements!!

Plan to mount the leadscrew



Points:

① Leadscrew will not generate much heat as it will be driven at slow speeds.

* Thrust Force on Bearings = $F_{preload} + F_{thrust}$
* Cutting Force (Radial Force) = $F_c/2$ on each side

Handwheel Selection

By conservation of energy

$$F l = \eta 2\pi \tau$$

F = Thrust force = 20 N

η = 0.3

l = 2 mm

τ = ?

$$\tau = \frac{F l}{2\pi \eta}$$

$$= \frac{20(2)}{2\pi(0.3)}$$

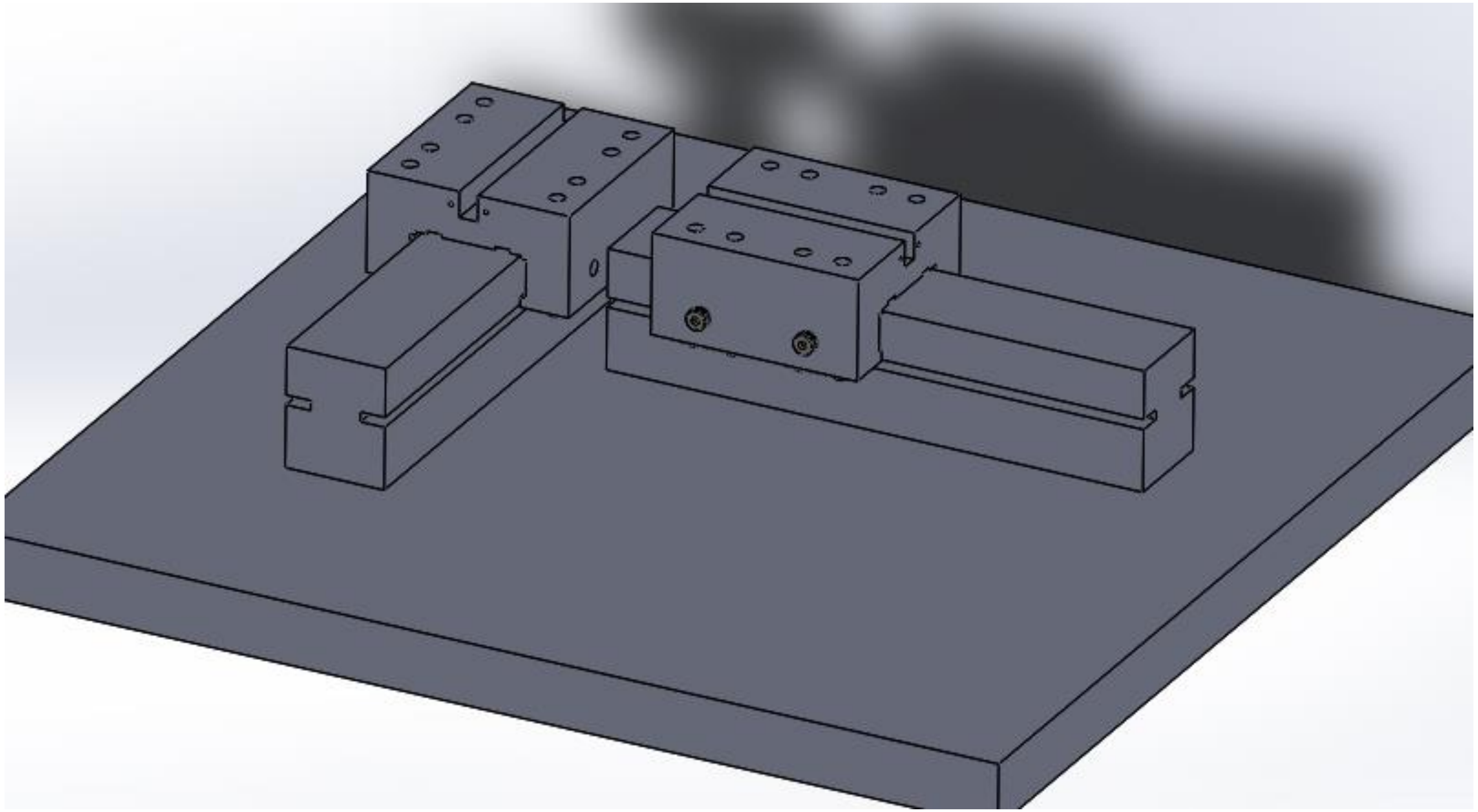
$$= 21.22 \text{ N mm.}$$

To make it easy for the person to rotate the wheel, a force of less than 0.5 N should be required.

$\therefore$ Diameter of Handwheel $> \frac{21.22}{0.5} = 42.44$ mm

Handwheel of 63 mm ordered from China!

Geometry of the Machine



Please refer to error budget spreadsheet for the updated error model for this system

Appendix

Calculations for Misalignment Budget

<u>Radial</u> <u>stiffness</u> (Varying w.r.t. a)

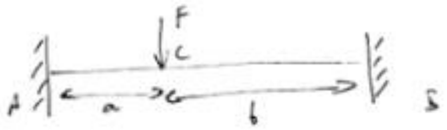
$$\delta_{a} = \cancel{2}\ \frac{Pa^3 b^3}{3EI\ell^3}$$

$$K_{radial} = P/\delta = \frac{3EI\ell^3}{a^3 b^3}$$

$$b = L - a.$$

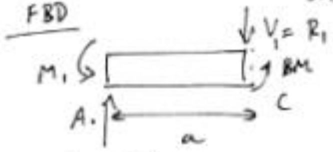
<u>MATLAB</u> <u>Plotting</u>

Bending Moment at point of load:



a+b=L.

FBD



$$R_1 = \frac{Pb^2}{\ell^3}(3a+b)$$

Actual BM, $R_1 a - M_1$

$$\frac{Pb^2}{\ell^2}(3a+b)a - \frac{Pab^2}{\ell^2}$$

$$\frac{Pb^2 a}{\ell^2}\left[\frac{3a+b}{\ell} - 1\right]$$

$$M_a = \frac{Pb^2 a}{\ell^2}\left[\frac{2a}{\ell}\right] = \frac{2Pb^2a^2}{\ell^3}$$

$$\sigma = \frac{M}{I}y = \frac{\left(\frac{2Pb^2a^2}{\ell^3}\right)(d/2)}{\pi/64\ d^4} \quad -(1)$$

Appendix

Matlab Code for Misalignment Analysis

```
yield=250;
E=200000;
d=5.5;
I=(3.142/64)*(d)^4;
y=d/2;
a=(10:1:115);
l=200;
leff=115;
b=l-a;
BMay=yield*I/y;
fay=BMay*(l^3)./(2.*(b.^2.*(a.^2)));
rstiffness=(3*E*I*(l^3))./(a.^3.*(b.^3));
Bucklingload=4*(pi^2)*E*I/(leff^2)
Shaftwhip=sqrt(rstiffness/0.08)*(30/pi)
theta1=0.0037
theta2=0.0065
theta3=0.009
delta1=theta1.*a;
delta2=theta2.*a;
delta3=theta3.*a;
Fexp1=delta1.*rstiffness;
Fexp2=delta2.*rstiffness;
Fexp3=delta3.*rstiffness;
%plot(a,fay,'b--',a,Fexp1,'r',a,Fexp2,'g',a,Fexp3,'black',a,rstiffness,'p');
plot(a,Shaftwhip)
```